MATHEMATICS
CLASS XII

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **29** questions divided into three sections **A**, **B,C** and **D**. Section **A** comprises **4** questions of **one mark** each, Section **B** comprises **8** questions of **two marks** each , Section **C** comprises 13 questions of **four marks** each and Section **D** comprises 6 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

SECTION – A

Q1 If $f : R \to R$ be defined by $f(x) = (3 - x^3)^{1/3}$, then find (f o f) (x). 1

Q2 If $\hat{a}, \hat{b}$ *and* $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $\left|2\hat{a} + \hat{b} + \hat{c}\right|$. 1

Q3 If A is a square matrix $A^2 = A$, then write the value of $(I + A)^2 - 3A$. 1

Q4 Let * be a binary operation on the set of all non – zero real numbers, given by a * b = $\frac{ab}{5}$ for all a, b $\in R - \{0\}$. Find the value of x, given that 2*(x *5) = 10 . 1

SECTION – B

Q5 Solve the equation $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$ 2

Q6 If A is 3×3 invertible matrix and $(5A)^{-1} = k A^{-1}$, find k . 2

Q7 Evaluate $\int \tan^8 x \sec^4 x dx$ 2

Q8 Verify mean value theorem for the function f(x) = (x – 3) (x – 6) (x – 9) in [3,5]. 2

Q9 The 2 vectors $\hat{j} + \hat{k}$ *and* $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A. 2

Q10 In 3 trials of a binomial distribution, the probability of exactly 2 successes is 9 times the probability of 3 successes. Find the probability of success in each trial. 2

Q11 Find the equation of a curve whose tangent at any point on it, different from origin, has slope $y + \frac{y}{x}$. 2

Q12 Water is dripping out from a conical funnel of semi – vertical angle $\frac{\pi}{4}$ at the uniform rate of $2cm^2$ /sec in the surface area, through a tiny hole at the vertex of the bottom. When the slant height of cone is 4 cm, find the rate of decrease of the slant height of water. 2

SECTION – C

Q13 Determine the values of a,b,c if the function 4

$$f\left(x\right)=\begin{cases}\dfrac{\sin\left(a+1\right)x+\sin x}{x} & if \quad x<0\\ c & if \quad x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & if \quad x>0\end{cases}$$ is continuous at x = 0

Q14 For what values of a,b and c, if any, does the function 4

$$f\left(x\right)=\begin{cases}ax^2+bx+c, & 0\le x\le 1\\ bx-c, & 1<x\le 2.\\ c, & x>2\end{cases}$$ become differentiable at x = 1 and x = 2?

Q15 To promote the making of toilets for women, an organization tried to generate awareness through i) house calls, ii) letters, and iii) announcements. The cost for each mode per attempt is given below: 4

i) Rs 50 ii) Rs 20 iii) Rs 40

The number of attempts made in three villages X,Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for the three villages separately , using matrices. Write one value generated by the organization in the society.

Q16 Find the equation of tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line 4x – 2y + 5 = 0. 4

Q17 Three numbers are given whose sum is 180 and the ratio between first two of them is 1 :2 . If the product of the number is greatest, find the numbers. 4

Q18 Evaluate : $\int\frac{\sin x-x\cos x}{x\left(x+\sin x\right)}dx$ or Evaluate : $\int e^{2x}\sin\left(3x+1\right)dx.$ 4

Q19 Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{x\left(2\log x+1\right)}{\sin y+y\cos y}$, given that $y=\frac{\pi}{2} when x=1.$ 4

OR

From the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

Q20 If $\vec{\alpha}=3\hat{i}+4\hat{j}+5\hat{k} and \vec{\beta}=2\hat{i}+\hat{j}-4\hat{k}$, then express $\vec{\beta}$ in the from $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ $and \vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. 4

Q21 A line passing through the point A with position vector $\vec{a}=4\hat{i}+2\hat{j}+2\hat{k}$ is parallel to the vector $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$. Find the length of the perpendicular drawn on this line from a point P with position vector $\vec{r}_1=\hat{i}+2\hat{j}+3\hat{k}$. 4

OR

let P (3,2,6) be a point in the space and Q be a point on the line

$\vec{r} = \hat{i} - \hat{j} + 2\hat{k} + \mu\left(-3\hat{i} + \hat{j} + 5\hat{k}\right)$,

then find the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x – 4y + 3z = 1.

Q22 In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed five times. If all the five times, the result was heads, find the probability that the selected coin had heads on both the sides. 4

Q23 Three cards are drawn successively with replacement from a well – shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution. 4

SECTION – D

Q24 Let A = $Q \times Q$ where Q is the set of all rational numbers, and * be a binary operation on A defined by (a,b) * (c,d) = (ac, b + ad) for (a,b), (c,d) $\in A$. Then find 6

(i) The defined element of * in A.

ii) Invertible elements of A, and hence write the inverse of elements (5,3) and (1/2 , 4) .

OR

If the function $f : \mathrm{R} \to R$ be defined as f(x) = 2x – 3 and g: $R \to R$ by g (x) = x^3 + 5 . then find the value of (fog)$^{-1}$ (x).

Q25 Using properties of determinants, Prove the following: 6

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x).$$

OR

Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by (x + y + z), and hence find the quotient.

Q26 Using the method of integration, find the area of the triangular region whose vertices are (2,-2), (4,3) and (1,2) . 6

Q27 Evaluate : $\int_1^3 \left(e^{2-3x} + x^2 + 1\right) dx$ as a limit of a sum.

OR

Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

Q28 Find the distance of point ($(-2,3,-4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane 4x + 12y – 3z + 1 = 0. 6

Q29 A box manufacturer makes large and small boxes from a large piece of cardboard. The large boxes require 4 sq m per box while the small boxes require 3 sq m per box. The manufacturer is required to make at least three large boxes and at most twice as many small boxes as large boxes. If 60 sq m of cardboard is in stock, and if the profits on the large and small boxes are Rs 3 and Rs 2 respectively, how many of each should be made in order to maximize the total profit? Formulate the above L.P.P. mathematically and then solve it graphically. 6

Ans : 1 (f o f) (x) = x 2 $\sqrt{6}$ 3 I 4 25 5 $x=\pm\frac{1}{12}$ 6 $\frac{1}{5}$ 7 $\frac{\tan^{11}x}{11}+\frac{\tan^{9}x}{9}+C$

8 $c=6-\sqrt{\frac{13}{3}}$ 9 $\frac{\sqrt{34}}{2}$ 10 p = ¼ 11 y = kx . e^x . 12 $\frac{\sqrt{2}}{4\pi}$ cm/s. 13 a = -3/2 , c = ½ and b is any non – zero real number.14 a = 0, b = 0, c = 0

15 x = Rs 30,000 ,y = Rs 23000, z = Rs 39000 16 48x – 24y = 23 17 40, 80,60

18 log|x|-log|x + sinx| + C or $\frac{2}{13}e^{2x}\sin(3x+1)-\frac{3}{13}e^{2x}\cos(3x+1)+C$

19 $y\sin y=x^2\log x+\frac{\pi}{2}$ or $\left(x^2+2xy\right)\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}+y^2+2xy=0$

20 $\beta_1=-\frac{3}{5}\hat{i}-\frac{4}{5}\hat{j}-\hat{k},\beta_2=\frac{13}{5}\hat{i}+\frac{9}{5}\hat{j}-3\hat{k}$ 21 $\sqrt{10}units$ or ¼ 22 $\frac{8}{9}$

23

X	0	1	2	3
P(X)	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

$\bar{x}=\frac{3}{4}$ 24 (i) (1,0) (ii) (a,b) $\in$ A $a\neq 0$ are invertible elements; $\left(\frac{1}{5},-\frac{3}{5}\right),(2-8)$ with OR

$(fog)^{-1}:\mathbb{R}\to\mathbb{R} given by (fog)^{-1}(x)=\sqrt[3]{\frac{x-7}{2}}$

26 area = $\frac{13}{2}sq.units$ 27 $\frac{32-e^{-7}+e^{-1}}{3}$ OR $\frac{\pi^2}{16}$ 28 $\frac{17}{2}units$ 29 no. of large boxes = 15, number of small boxes = 0 and maximum profit = Rs 45.