ॐ ॐ

UNIVERSAL EDUCATION CENTRE

JAYANT SHARMA (94145-37474)

www.uecj4u.hpage.co.in

Time 3 hrs **Maths 12th** **M.M. 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of 29 questions divided into three sections-A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted.*

SECTION–A

1. If $f(x) = x + 7$ and $g(x) = x - 7, x \in R$, find $(fog)(7)$
2. Evaluate : $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$
3. Find the value of x and y if : $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$
4. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$
5. Find the cofactor of a_{12} in the following: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
6. Evaluate: $\int \frac{x^2}{1+x^3}\,dx$
7. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$
8. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

9. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

10. For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other?

SECTION–B

11. (*i*) Is the binary operation defined on set N, given by $a * b = \frac{a+b}{2}$ for all $a, b \in N$, commutative?

(*ii*) Is the above binary operation associative?

12. Prove the following:

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

13. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$.

Express A as sum of two matrices such that one is symmetric and the other is skew symmetric.

OR

If $A = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$, verify that $A^2 - 4A - 5I = 0$

14. For what value of k is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} 2x + 1 & ; x < 2 \\ k & ; x = 2 \\ 3x - 1 & ; x > 2 \end{cases}$$

15. Differentiate the following with respect to x : $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

16. Find the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \frac{\pi}{4}$

17. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$

18. Solve the following differential equation:

$$(x^2 - y^2)\, dx + 2xy\, dy = 0$$

given that $y = 1$ when $x = 1$

OR

Solve the following differential equation:

$$\frac{dy}{dx} = \frac{x(2y - x)}{x(2y + x)}, \quad \text{if } y = 1 \text{ when } x = 1$$

19. Solve the following differential equation : $\cos^2 x \frac{dy}{dx} + y = \tan x$

20. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} . \vec{c} = 3$.

OR

If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$, show that the angle between $\vec{a}$ and $\vec{b}$ is $60°$.

21. Find the shortest distance between the following lines :

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

OR

Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3).

22. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.

SECTION–C

23. Using properties of determinants, prove the following :

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$

24. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$.

25. Using integration find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

26. Evaluate: $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} dx$

27. Find the equation of the plane passing through the point $(-1, -1, 2)$ and perpendicular to each of the following planes:

$2x + 3y - 3z = 2$ and $5x - 4y + z = 6$

OR

Find the equation of the plane passing through the points (3, 4, 1) and (0, 1, 0) and parallel to the line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$

28. A factory owner purchases two types of machines, *A* and *B* for his factory. The requirements and the limitations for the machines are as follows :

Machine	Area occupied	Labour force	Daily output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available, and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output?

29. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver.