CLASS XII
SAMPLE PAPER
MATHS

Probability

1. The probability of a man hitting of target is1/4. If he fires 7 times , what is the probability of hitting the target at least two times.
2. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the mean and variance and S.D. of the number of queens.
3. Find the probability distribution of number of doublets in three throws of a pair of dice.
4. A and B throw a die alternatively till one of them gets 6 and wins the game. Find their respectively probabilities , if A starts the game.
5. In a game , a person is paid Rs. 5 if he gets all heads or all tails, when three coins are tossed , and he will paid Rs. 3 if one or two head shows. What can be expected to win on the average per game.
6. How many times must a man toss a fair coin , so that the probability of having at least one head is more than 80%.
7. A biased coin is twice as likely to show an even number as an odd number. The die is rolled three times. If occurrence of an even number is considered a success , then write the probability

distribution number of successes. Also find the mean number of successes.

8. Two bags A and B contain 4 white 3 black balls and 2 white and 2 black balls respectively. From bag A , two balls are drawn at random and transferred to bag B. A ball is then drawn from bag B and is found to be a black ball. What is the probability that the transferred ball were 1 white and 1 black.
9. In a binomial distribution , the sum and the product of the mean and variance are 25/3 and 50/3 respectively. Find the distribution.
10. A police man fires 6 bullets on a burglar. The probability that the burglar will be hit by 0.6 . What is the probability that the burglar is still unhurt ?
11. In a binomial distribution , prove that mean > variance
12. Find the probability distribution of the number of sixes in three tosses of a die.
13. A card from a pack of 52 cards is lost. From the remaining cards of a pack , two cards are drawn and are found to be both spades . Find the probability of the lost card being a spade .
14. A man is known to speak the truth 3 out of 4 times . He throws a die and reports that it is a six. Find the probability that it is actually a six.
15. A doctor is to visit a patient. From past experience , it is known that the probabilities that he will come by train , bus , scooter or by car are respectively 3/10 , 1/5 , 1/10 and 2/5 . The probabilities that he will be late are 1/4 ,1/3 and 1/12 , if he comes

by train , bus and scooter respectively ; but he comes by car, he will be late . When he arrives , he is late . What is the probability that he has come by train ?

16. Two numbers are selected at random from the integers 1to 9.If the sum is even , find the probability that both the numbers are odd .

17. Two unbiased dice are drawn . Find the probability that the sum is 8 or greater if 4 appears on the first die .

18. A die is rolled . If the outcome is an odd number , what is the probability that it is prime ?

19. Obtain the binomial distribution whose mean is 10 and the standard deviation is $2\sqrt{2}$.

20. If on an average , out of 10 ships , one gets drowned then what is the probability that out of 5 ships at least 4 reach the shore safely ?

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- ## Lagrange's mean value and Rolle's theorem

1. Verify Rolle's theorem for each of the following functions:
   (i) $f(x) = \sin 2x$ in $[0,\pi/2]$ (ii) $f(x) = (\sin x - \sin 2x)$ in $[0,\pi]$
2. Verify Rolle's theorem for each of the following functions:
   (i) $f(x) = x^3 + 3x^2 - 24x - 80$ in $[-4,5]$
   (ii) $f(x) = \sqrt{(1 - x^2)}$ in $[-1,1]$
   (iii) $f(x) = (x - 1)(x - 2)(x - 3)$ in $[1,3]$

3. Show that $f(x) = x (x - 5)^2$ satisfies Roll's theorem on [0,5] and that value of c is (5/3)
4. Using Rolle's theorem ,find the point on the curve $y = x (x - 4)$, $x \in [0,4]$ where the tangent is parallel to x-axis.
5. If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5$ on [1,3] with $c= [2 + 1/\sqrt{3}]$, find the values of a and b.
6. Discuss the applicability of Rolle's theorem on : $f(x) = \tan x$ in$[0, \pi]$
7. Verify Rolle's theorem for the function $f(x) = (x - a)^m (x - b)^n$ in the interval [a,b] ,where m and n are positive integers.
8. If $f(x) = x(1 - \log x)$, where $x>0$, show that $(a - b) \log c = b(1 - \log b) - a(1 - \log a)$, where $0<a<c<b$
9. Find the points on the curve $y = x^3 - 3x$, where the tangent to the curve is parallel to the chord joining (1,-2) and (2,2).
10. Verify L.M.V. for the following functions $f(x) = \tan^{-1} x$ on [0,1].
11. Verify L.M.V. for the following functions $f(x) = \log x$ on [1,e]
12. Verify L.M.V. for the following functions $f(x) = 2x^2 - 3x + 1$ on [1,3]
13. Verify L.M.V. for the following functions $f(x) = e^x$ on [0,1]
14. Show that L.M.V. is not applicable to $f(x) = 1/x$ on [-1,1]
15. Find c of the mean value theorem for the functions : $f(x) = 2x^2 - 10x + 29$ in [2,7]

16. Verify the hypothesis and conclusion of L.M.V. for the function f(x) = $1/(4x-1)$, $1 \le x \le 4$.

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17. Verify Rolle's theorem for each of the following functions:
(i) f(x) = sin2x in [0,π/2] (ii) f(x) = (sinx – sin2x) in [0,π]

18. Verify Rolle's theorem for each of the following functions:
(i) $f(x) = x^3 + 3x^2 - 24x - 80$ in [-4,5]
(ii) $f(x) = \sqrt{(1-x^2)}$ in [-1,1]
(iii) f(x) = (x -1) (x -2) (x -3) in [1 ,3]

19. Show that $f(x) = x(x-5)^2$ satisfies Roll's theorem on [0,5] and that value of c is (5/3)

20. Using Rolle's theorem ,find the point on the curve y = x (x - 4), x ϵ [0,4] where the tangent is parallel to x-axis.

21. If Rolle's theorem holds for the function
$f(x) = x^3 + bx^2 + ax + 5$ on [1,3] with c= [$2 + 1/\sqrt{3}$], find the values of a and b.

22. Discuss the applicability of Rolle's theorem on :
f(x) =tanx in[0, π]
23.Verify Rolle's theorem for the function
$f(x) = (x-a)^m (x-b)^n$ in the interval [a,b] ,where m and n are positive integers.

23. If f(x) = x(1 –logx), where x>0 , show that
(a –b) logc = b(1 –logb) – a(1 – loga), where 0<a<c<b

24. Find the points on the curve $y = x^3 - 3x$, where the tangent to the curve is parallel to the chord joining (1,-2) and (2,2).
25. Verify L.M.V. for the following functions $f(x) = \tan^{-1} x$ on [0,1].
26. Verify L.M.V. for the following functions $f(x) = \log x$ on [1,e]
27. Verify L.M.V. for the following functions $f(x) = 2x^2 - 3x + 1$ on [1,3]
28. Verify L.M.V. for the following functions $f(x) = e^x$ on [0,1]
29. Show that L.M.V. is not applicable to $f(x) = 1/x$ on [-1,1]
30. Find c of the mean value theorem for the functions : $f(x) = 2x^2 - 10x + 29$ in [2,7]
31. Verify the hypothesis and conclusion of L.M.V. for the function $f(x) = 1/(4x - 1)$, $1 \le x \le 4$.

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Indefinite Integration

1. Integrate : $\int(\log x / x^2)dx$
2. Integrate : $\int (\sin x \cos x / 1 + \sin^4 x)\, dx$
3. Integrate : $\int \sqrt{\tan x}\, dx$
4. Integrate : $\int e^{2x} \sec 2x\, (1 + \tan 2x)\, dx$
5. Integrate : .$\int \log(\log x) + 1/(\log x)^2\, dx$
6. Integrate : $\int \cos^4 x dx$
7. Integrate : $\int [(x^4 - x)^{1/4}] / x^5\, dx$
8. Integrate : $\int \sqrt{\tan x}\, (1 + \tan^2 x)\, dx$

9. Integrate : $\int \sin^{-1}(\cos x)\,dx$

10.Integrate : $\int (2x + 1)/(x^3 - 1)\,dx$

11. Integrate : $\int [(x + 3) / \sqrt{x^2 + 4x + 5}]\,dx$

12. Integrate : $\int \left[\dfrac{(xe^x + 1)(e^x + \log x)^2}{x}\right] dx$

13. Integrate : $\int dx / [3x^2 + 13x + 10]$

14. Integrate : $\int [x^2 + 4x)dx / [x^3 + 6x^2 + 5]$

15. Integrate : $\int \sqrt{[\sin(x - a) / \sin(x + a)]}\,dx$

16. Integrate : $\int x^2 \tan^{-1} x\,dx$

17. Integrate : $\int [(2\sin 2x - \cos x) / (6 - \cos^2 x - 4\sin x)]\,dx$

18. Integrate : $\int [\sin 2x / (a + b\cos x)^2]\,dx$

19. Integrate : $\int [\sqrt{\tan x} + \sqrt{\cot x}]\,dx$

20. Integrate : $\int e^x \sec x (1 + \tan x)\,dx$

21. Integrate : $\int dx/[x + 2\sqrt{(x + 3)}]$

22. Integrate : $\int \sin 2x \cos 2x\,dx / (9 - \cos^4 2x)$

23. Integrate : $\int (\log x)^3\,dx / x$

24. Integrate : $\int [\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}]\,dx / [\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}]$

25. Integrate : $\int dx / \sin(x - a)\sin(x - b)$

26. Integrate : $\int dx / (x - \sqrt{x})$

27. Integrate : $\int (1 + \cot x)\, dx / x + \log(\sin x)$

28. Integrate : $\int dx / (x - 1)\sqrt{(2x + 3)}$

29. Integrate : $\int \sec^2 x\, dx / \sqrt{(1 - \tan^2 x)}$

30. Integrate : $\int (\tan x + \tan^3 x)\, dx / (1 + \tan^3 x)$

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Error and Approximations

1.Find the approximate change in the surface area of a cube of side x cm caused by decreasing the side by 1%.

2. If the radius of a sphere is measured as 7 m with an error of 0.02 m , find the approximate error in calculating its volume.

3. If $y = x^4 - 10$ and if x changes from 2 to 1.99, what is the approximate change in y ?

4. Find the value by using differentials :-

(i) $(25.3)^{1/2}$

(ii) $(o.o37)^{½}$

(iii) $(242)^{1/5}$

(iv) $(3.968)^{3/2}$

(v) $(36.5)^{1/2}$

5. Find the approximate value of $f(3.02) = 3x^2 + 5x + 3$

6. A spherical balloon has an initial radius of 10 cm. Find the approximate decrease in its volume if it is compressed to a spherical balloon with radius 9.9 cm.

7. Calculate the value of $(31.9)^{1/5}$ by using differentials.

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