CLASS XII
SAMPLE PAPER
MATHS

SOME IMPORTANT QUESTIONS ON CHAPTERS 3, 4, 5 AND 6

Section- A (One mark each)

1) Find the value of **a** for which $\begin{bmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ is non – invertible.
2) If A is a square matrix such that $|A|$= 2, write the value of $|A^3|$
3) Find the value of the determinant $\begin{vmatrix} x & 2 & y+z \\ y & 2 & z+x \\ z & 2 & x+y \end{vmatrix}$
4) A continuous function is defined as $f(x) = \begin{cases} \frac{2x}{\sin x}, & x < 0 \\ \frac{\tan ax}{x}, & x > 0 \\ b, & x = 0 \end{cases}$. Find $a^2 + b^2$
5) Let $\lim_{x\to 3-0} f(x)$ = a, $\lim_{x\to 3+0} f(x)$ = b and a + b = 4. Find the value of f(3) if f(x) is continuous at x = 3.
6) If y = $\frac{1}{1+x^{m-n}} + \frac{1}{1+x^{n-m}}$, find the value of $\frac{dy}{dx}$
7) Find the interval in which f(x) = $|x|$ is a decreasing function.
8) Find the angle between tangents to the curve where it cuts x – axis.
9) Find the value of x where f(x) = x log x has local minimum.
10) If $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ is finite and unique, then write the value of $\lim_{x\to a} f(x)$

Section- B (Four marks each)

11) If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find k if $A^2 = kA - 2I$
12) If A = $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$; B = $\begin{bmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{bmatrix}$, then show that AB is a zero matrix, provided ($\theta - \varphi$) is an odd multiple of $\frac{\pi}{2}$

13) Using properties of determinants , prove that :

$$\begin{vmatrix} -a(-a^2+b^2+c^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(a^2-b^2+c^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$$

14) If 2s = a + b + c, show that $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-a)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$

15) Determine the values of a, b, c for which the function f defined by

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \\ c, & x = 0 \end{cases}$$

is continuous at x = 0

16) Find the values of a and b such that the function f(x) defined by

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \le x \le \frac{\pi}{4} \\ 2x\cot x + b, & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x, & \frac{\pi}{2} < x \le \pi \end{cases}$$

is continuous for all x in $0 \le x \le \pi$

17) If $y = e^{ax}\sin bx$, prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$

18) Verify Rolle's theorem for the function $f(x) = x(x+3)e^{-\frac{x}{2}}$

19) Prove that $\frac{x}{1+x} < \log(1+x) < x$, for $x > 0$

20) Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$is i) strictly increasing, ii) strictly decreasing

21) Show that the curves $x^3 - 3xy^2 = a$ and $3x^2y - y^3 = b$ cut each other orthogonally, where a and b are constants.

22) Find the angle of intersection of the curves $y^2 = 4ax$ and $x^2 = 4by$.

Section- C (six marks each)

23) For matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$, hence find A^{-1}

24) If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

25) A given quantity of metal is to be cast into a half cylinder i.e. with a rectangular base and semi-circular ends. Show that in order that the total surface area may be minimum the ratio of the height of the cylinder to the diameter of the semi-circular ends is $\frac{\pi}{\pi+2}$

26) If the length of three sides of a trapezium other than base are equal to 10 cm, then find the area of trapezium when it is maximum.

27) Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

28) Prove that a rectangle of maximum area that can be inscribed in an equilateral triangle of side b will be $\frac{\sqrt{3}\,b^2}{8}$

29) The fuel charges for running a train are proportional to the square of the speed generated in mile/h and cost Rs. 48 per hour at 16 miles/h. Show that the most economical speed of the train if the fixed charges i.e. salaries etc amount to Rs. 300 per hour is 40 mile/h