Series : PTS/9

Code No. 16/10/9

Candidates must write the Code on the title page of the answer-book.

Roll No. 2 1 6 1 2

PLEASURE TEST SERIES XII – 09

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. If $f : R \to R$, $g : R \to R$ be defined by $f(x) = \frac{3x-7}{8}$, $g(x) = \frac{8x+7}{3}$ then, find $fog(7)$.

Q02. Find the sum of the cofactors of a_{12} and a_{21} in $\begin{vmatrix} 2 & 3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

Q03. Write the degree of the differential equation $\left(\frac{d^2s}{dx^2}\right)^3 + \left(\frac{ds}{dx}\right)^2 + \sin\left(\frac{ds}{dx}\right) + 1 = 0$.

Q04. If A is 3×4 matrix and B is a matrix such that $A^T B$ and BA^T are both defined. Then write the order of matrix B.

SECTION B

Q05. Draw the neat & labeled graph for $\sin^{-1} x$.

Q06. Write the value of λ, if the vectors $\vec{a} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + 2\hat{k}$ are orthogonal vectors.

Q07. Check if (3,–5, 1), (–1, 0, 8) and (7,–10,–6) are collinear points or not.

Q08. Write the value of $\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$.

Q09. Using MV Theorem, find a point on the curve $y = (x-3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

Q10. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane.

Q11. If A is a square matrix of order 3, then write the value of adj.(adj.A) where $|A| = 14$.

Q12. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X).

SECTION C

Q13. Find x, y, z if $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ satisfies $A' = A^{-1}$.

OR Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$. Verify that $(A^{-1})^{-1} = A$.

Q14. Simplify the expression : $\frac{1}{2}\cos^{-1}\left(\frac{5\cos x + 3}{5 + 3\cos x}\right)$.

Q15. Discuss the continuity and differentiability of $|2x + 9|$.

OR Discuss the differentiability of $f(x) = [x]$ at $x = 2$. Here [x] is greatest integer function.

Q16. If x = a sin2t (1 + cos2t) and y = b cos2t (1 – cos2t), show that $\left(\frac{dy}{dx}\right)_{at\ t=\pi/4} = \frac{b}{a}$.

Q17. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$. How do you think that together we can make a difference in the society about importance of trees?

OR A plane is ascending vertically at the rate of 100 kmph. If the radius of the earth is R km, how fast is the area of the earth (visible from the plane), increasing at 3 minutes after it started ascending? Given that the visible area A at height h is given by $A = 2\pi R^2 \frac{h}{R+h}$.

Q18. Find : $\int \frac{\cos^2 x\, dx}{1+\tan x}$.

Q19. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$ then, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

Q20. Find the particular solution of the differential equation : $(2y+x)dy - (2y-x)dx = 0,\ y(1) = 1$.

Q21. Find a point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of $3\sqrt{2}$ units from (1, 2, 3).

Q22. If P (A) = 0.4, P (B) = p, $P(A \cup B) = 0.6$ and A and B are given to be independent events, find the value of p.

Q23. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes. Hence find the variance of the distribution.

SECTION D

Q24. Let X be a non-empty set. Consider a binary operation ∗ : P(X) × P(X) → P(X) given by A ∗ B = A ∩ B ∀A, B in P(X), where P(X) is the power set of X.
Find the identity element for this operation. Also show that X is the only invertible element in P(X) with respect to the operation ∗.

Q25. Using properties of determinants, prove that : $\begin{vmatrix} a+b+nc & na-a & nb-b \\ nc-c & b+c+na & nb-b \\ nc-c & na-a & c+a+nb \end{vmatrix} = n(a+b+c)^3$.

OR Prove that : $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$.

Q26. Find the area of the region bounded by the curve $y = x^2 + x$, x-axis and the line x = 2 and x = 5.

OR Using the method of integration find the area bounded by the curve |x| + |y| = 1.

Q27. Evaluate : $\int_{\pi/2}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$. **OR** Evaluate : $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$.

Q28. Find the equation of the plane which passes through the points (3, 4, 1) and (0, 1, 0) and is parallel to $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$. Also find the intercepts cut-off by this plane on the axes.

Q29. A farmer mixes two brands P and Q of cattle feed. Brand P, costing ₹250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing ₹200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? Find the minimum cost of the mixture per bag?

SOLUTIONS & MARKING SCHEME for PTS - 09 [2016 - 2017]

SECTION A

Q01. We have $fog(7) = f[g(7)] = f\left(\frac{8\times 7+7}{3}\right) = f(21) = \frac{3\times 21-7}{8} = 7$.

Q02. Cofactor of a_{12} = – [(6)(–7) – (4)(1)]= 46 and, cofactor of a_{21} = – [(3)(–7) – (5)(5)]= 46.
Therefore, the required sum is 92.

Q03. Degree = 3.

Q04. Order of B = 4×3.

SECTION B

Q05. See NCERT Part I Chapter 02

Q06. 2

Q07. Points are collinear.

Q08. Let $I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ $\Rightarrow I = \int_{-\pi/4}^{\pi/4} \frac{dx}{2\cos^2 x} = \frac{1}{2}\int_{-\pi/4}^{\pi/4} \sec^2 x dx$

$\Rightarrow I = \frac{1}{2}\left[\tan x\right]_{-\pi/4}^{\pi/4}$ $\Rightarrow I = \frac{1}{2}\left[\tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)\right] = \frac{1}{2}[1+1] = 1$.

Q09. (7/2, 1/4).

Q10. $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k}) = 29$

Q11. Using adj.(adj.A) = $|A|^{n-2}A$, we get : $adj.(adj.A) = 14^{3-2}A = 14A$.

Q12.

X	P(X)	X P(X)
0	30%	0
1	70%	70%
		70%

Therefore, $E(X) = \sum X\,P(X) = 70\%$ or 0.7

SECTION C

Q13. Given that $A' = A^{-1}$ $\Rightarrow AA' = AA^{-1} = I$ [Pre-multiplying both sides by A
Now complete yourself.
OR Do yourself.

Q14. Let $y = \frac{1}{2}\cos^{-1}\left(\frac{5\cos x+3}{5+3\cos x}\right) = \frac{1}{2}\cos^{-1}\left(\frac{5\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+3}{5+3\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}\right)$

$$\Rightarrow y = \frac{1}{2}\cos^{-1}\left(\frac{5\left(1-\tan^2\frac{x}{2}\right)+3\left(1+\tan^2\frac{x}{2}\right)}{5\left(1+\tan^2\frac{x}{2}\right)+3\left(1-\tan^2\frac{x}{2}\right)}\right) = \frac{1}{2}\cos^{-1}\left(\frac{(5+3)-(5-3)\tan^2\frac{x}{2}}{(5+3)+(5-3)\tan^2\frac{x}{2}}\right)$$

$$\Rightarrow y = \frac{1}{2}\cos^{-1}\left(\frac{4-\tan^2\frac{x}{2}}{4+\tan^2\frac{x}{2}}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1-\left(\frac{1}{2}\tan\frac{x}{2}\right)^2}{1+\left(\frac{1}{2}\tan\frac{x}{2}\right)^2}\right) = \frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right) = \tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right).$$

Q15. Continuous but not differentiable at x = –9/2
OR Here $f(x) = [x]$ $\therefore f(2) = [2] = 2$

$Lf'(2) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{[x] - 2}{x - 2}$ $\left[\begin{array}{l}\text{Put } x = 2 - h. \\ \text{As } x \to 2 \Rightarrow h \to 0\end{array}\right.$

$\Rightarrow \quad = \lim_{h \to 0} \frac{[2-h]-2}{-h} = \lim_{h \to 0} \frac{1-2}{-h} = \lim_{h \to 0} \frac{-1}{-h} = \lim_{h \to 0} \frac{1}{h} = \frac{1}{0}$, which is not defined.

Hence f is not differentiable at x = 2.

Q16. We have x = a sin2t (1 + cos2t) = 2a sin2t cos^{2}t $\Rightarrow \frac{dx}{dt} = 2a\left(-\sin^2 2t + 2\cos^2 t \cos 2t\right)$

And y = b cos2t (1 – cos2t) = 2b cos2t sin^{2}t $\Rightarrow \frac{dy}{dt} = 2b\left(\cos 2t \sin 2t - 2\sin^2 t \sin 2t\right)$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{2b\left(\cos 2t \sin 2t - 2\sin^2 t \sin 2t\right)}{2a\left(-\sin^2 2t + 2\cos^2 t \cos 2t\right)}$

So, $\left(\frac{dy}{dx}\right)_{at\ t = \pi/4} = \frac{b\left(\cos 2(\pi/4) \sin 2(\pi/4) - 2\sin^2 (\pi/4) \sin 2(\pi/4)\right)}{a\left(-\sin^2 2(\pi/4) + 2\cos^2 (\pi/4) \cos 2(\pi/4)\right)} = \frac{b}{a}$. Hence proved.

Q17. Here $y^2 = 4ax$ …(i) and $xy = c^2$ …(ii) $\Rightarrow \frac{dy}{dx} = \frac{2a}{y}$ and $\frac{dy}{dx} = -\frac{y}{x}$

If curves are cutting each other at right angles then, $\frac{2a}{y}\left(-\frac{y}{x}\right) = -1 \Rightarrow x = 2a$...(A)

By (i), when x = 2a, $y^2 = 4a \times 2a \therefore y = 2a\sqrt{2}$ …(B)

Replacing value of x and y from A and B in (ii), we get : $2a \times 2a\sqrt{2} = c^2$

Squaring both sides, we get : $32a^4 = c^4$.

OR We have $A = 2\pi R^2 \frac{h}{R+h}$ $\Rightarrow A = 2\pi R^2 \left(1 - \frac{R}{R+h}\right)$

$\Rightarrow \frac{dA}{dt} = 2\pi R^2 \left(0 + \frac{R}{(R+h)^2} \times \frac{dh}{dt}\right)$ $\Rightarrow \frac{dA}{dt} = \frac{2\pi R^3}{(R+h)^2} \times \frac{dh}{dt}$ …(i)

Since the plane covers 100 km in 60 minutes so, it shall cover $\left(\frac{100}{60} \times 3\right) = 5\,\text{km}$ in 3 minutes.

By (i), $\frac{dA}{dt} = \frac{2\pi R^3}{(R+5)^2} \times 100 = \frac{200\pi R^3}{(R+5)^2}$ km^2 / hr.

Q18. Obtain $I = \int \frac{\cos^3 x\, dx}{\sin x + \cos x} = \frac{1}{2}\int \frac{(\cos^3 x + \sin^3 x) + (\cos^3 x - \sin^3 x)}{\sin x + \cos x} dx$

i.e., $I = \frac{x}{2} + \frac{1}{4}\log|\sin x + \cos x| + \frac{1}{8}(\cos 2x + \sin 2x) + k$.

Q19. $\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$ [See Solutions of CBSE 2013 Delhi (set 2)]

Q20. We have $(2y + x)dy - (2y - x)dx = 0 \Rightarrow \frac{dy}{dx} = \frac{2y - x}{2y + x}$. Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So we get : $v + x\frac{dv}{dx} = \frac{2vx - x}{2vx + x}$

$\Rightarrow \int \frac{(2v+1)dv}{2v^2 - v + 1} = -\int \frac{dx}{x} \Rightarrow \frac{1}{2}\int \frac{(4v-1)dv}{2v^2 - v + 1} + \frac{3}{2}\int \frac{dv}{2v^2 - v + 1} = -\int \frac{dx}{x}$

$$\Rightarrow \frac{1}{2}\log|2v^2 - v + 1| + \frac{3}{4}\int \frac{dv}{\left(v-\frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\log\left|2\left(\frac{y^2}{x^2}\right) - \frac{y}{x} + 1\right| + \frac{3}{4}\times\frac{4}{\sqrt{7}}\tan^{-1}\left(\frac{4v-1}{\sqrt{7}}\right) = -\log|x| + C$$

$$\Rightarrow \log\left|2y^2 - xy + x^2\right| - 2\log|x| + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{x\sqrt{7}}\right) = -2\log|x| + 2C$$

$$\Rightarrow \log\left|2y^2 - xy + x^2\right| + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{x\sqrt{7}}\right) = K, \text{ where } K = 2C.$$

Given that y = 1 when x = 1 so, $\log 2 + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{3}{\sqrt{7}}\right) = K$

Hence the solution is : $\Rightarrow \log\left|2y^2 - xy + x^2\right| + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{x\sqrt{7}}\right) = \log 2 + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$

Q21. Let P(1, 2, 3). Coordinates of random point on $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} =$ m is M(3m –2, 2m – 1, 2m + 3).

So PM = $3\sqrt{2} \Rightarrow m = 0, 30/17$. $\therefore$ Required points : $(-2, -1, 3)$ and $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$.

Q22. See Mathematicia Vol. 2 *by* O.P. Gupta. Ans. 1/3.

Q23. Let X : number of doublets in 4 throws of dice. So X = 0, 1, 2, 4.

X	0	1	2	3	4
P(X)	625/1296	500/1296	150/1296	20/1296	1/1296

Now mean $= \sum X\ P(X) = 0\times\frac{625}{1296} + 1\times\frac{500}{1296} + 2\times\frac{150}{1296} + 3\times\frac{20}{1296} + 4\times\frac{1}{1296} = \frac{864}{1296} = \frac{2}{3}$.

Varianc $= \sum X^2 P(X) - (\text{Mean})^2 = \left(0\times\frac{625}{1296} + 1\times\frac{500}{1296} + 4\times\frac{150}{1296} + 9\times\frac{20}{1296} + 16\times\frac{1}{1296}\right) - \frac{4}{9}$

$\Rightarrow \qquad = \frac{1296}{1296} - \frac{4}{9} = \frac{5}{9}$.

SECTION D

Q24. It is known that $* : P(X)\times P(X) \to P(X)$ is defined as $A*B = A\cap B\ \forall\ A, B \in P(X)$.
Let X be the identity element. So $A*X = A = X*A\ \forall\ A \in P(X)$
$\Rightarrow A\cap X = A = X\cap A\ \forall\ A \in P(X)$. Therefore, X is the identity element for the given binary operation *.
Now, an element $A \in P(X)$ is invertible if $\exists$ an element $B \in P(X)$ s.t. $A*B = X = B*A$.
That is, $A\cap B = X = B\cap A$, which is possible only if $A = B = X$.
Thus, X is the only invertible element in P(X) w. r. t. the given operation *.

Q25. See Mathematicia Vol.1 *by* O.P. Gupta **OR** See Mathematicia Vol.1 *by* O.P. Gupta.

Q26. Construct a labeled and clean diagram. Required area = $\int_2^5 (x^2 + x)dx = \frac{99}{2}$ Sq.units.

OR See Mathematicia Vol.1 *by* O.P. Gupta.

Q27. See Mathematicia Vol.1 *by* O.P. Gupta **OR** See Mathematicia Vol.1 *by* O.P. Gupta.

Q28. Let A, B, C be d.r.'s of normal of the plane passing through (3, 4, 1) and (0, 1, 0).
So the plane is A (x – 3) + B(y – 4) + C(z – 1) = 0 …(i)
As (i) passes through (0, 1, 0) i.e., – 3A – 3B – C = 0 i.e., 3A + 3B + C = 0 …(ii)

Also plane is parallel to $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$ therefore, 2A + 7B + 5C = 0 …(iii)

By (ii) and (iii), we get $\frac{A}{8}=\frac{B}{-13}=\frac{C}{15}$ i.e., d.r.'s of normal are 8, –13, 15.

Replacing these values in (i) : $8(x-3)-13(y-4)+15(z-1)=0$ i.e., $8x-13y+15z+13=0$.

Also, $8x-13y+15z=-13 \Rightarrow \frac{x}{-13/8}+\frac{y}{1}+\frac{z}{-13/15}=1$

$\therefore$ x intercept $=-13/8$, y intercept $=1$, z intercept $=-13/15$.

Q29. Let the number of bags of Type P and Type Q be x and y respectively.

To minimize : Z = ₹(250x + 200y)

Subject to constraints : $x \ge 0,\ y \ge 0,\ 3x+1.5y \ge 18,\ 2.5x+11.25y \ge 45,\ 2x+3y \ge 24$.