MODEL PRACTICE TEST PAPER - IV
MATHEMATICS
CLASS 12 - CBSE 2011

Time : 3 hrs

Max. Marks: 100

General Instructions:

1. All questions are compulsory
2. The question paper consists of 29 questions divided into three sections A,B and C. Section A contains 10 questions of 1 mark each, Section B contains 12 questions of 4 marks each and section C contains 07 questions of 6 marks each.

Section – A
(Questions 1 – 10 carry one mark each)

1. Find the position vector of the midpoint of the line segment joining the points A($5\hat{\imath} + 3\hat{\jmath}$) and B($3\hat{\imath}$-$\hat{\jmath}$)
2. Evaluate $\int sec^2(7-x)dx$
3. Write the value of p for which $\vec{a}$ = $3\vec{\imath}$ + $2\vec{\jmath}$ + 9 $\vec{k}$ and $\vec{b}$ = $\imath$ + p$\hat{\jmath}$ + $3\hat{k}$ are parallel vectors
4. Evaluate $\int sin7x\ sinx\ dx$
5. Using the property of determinant prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$
6. Evaluate $\int sin^4 xdx$
7. Form the differential equation corresponding to y^2=a(b-x^2) where a and b a re arbitrary constants
8. If $\hat{a}$ and $\hat{b}$ are two unit vectors and θ is the angle between them, show that sin$\frac{\theta}{2}$ = $\frac{1}{2}$|$\hat{a}$-$\hat{b}$|
9. Find the principle value of $\tan^{-1}(-1)$
10. Evaluate $\int \frac{1}{3+2sinx+cosx}$

Section – B
(Questions 11 – 22 carry four marks each)

11. Integrate $\int_0^{\frac{\pi}{2}}(\sqrt{tanx} + \sqrt{cotx})$ dx
12. Solve for X: $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$ or, Prove that $2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \tan^{-1}(\frac{31}{17})$
13. If x^y+y^x=a^b, find $\frac{dy}{dx}$ or If $\sqrt{(1-x)^2} + \sqrt{(1-y)^2}$ = a(x-y), prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$
14. Form the differential equation of the family of circles having radii 3.
15. For the curve y = $4x^3$-$2x^5$, find all points at which the tangent passes through the origin.

    Or,

    Show that normal at any point θ to the curve x=acosθ+a$\theta sin\theta$ and y=asin$\theta - a\theta cos\theta$ is at constant distance from the origin.

    Or,

    Find the intervals in which the function given by $x^3 + \frac{1}{x^3}$, x≠ 0 is (i) increasing (ii) decreasing
16. Using properties of determinants, show that $\begin{bmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{bmatrix}$ = (1+a^2+b^2+c^2)

    Or,

    Using the properties of determinants show that $\begin{bmatrix} (b+c)^2 & ab & ca \\ ab & (a+b)^2 & bc \\ ac & bc & (a+b)^2 \end{bmatrix}$ = 2abc(a+b+c)^3
17. Integrate $\int e^x\{\frac{x^2+1}{(x+1)^2}\}$ or,

    Find all the points of discontinuity of the function f(x) = [x^2] on (1,2), where [.] denotes the greatest integer function.
18. Find whether the lines $\vec{r}$ = ($\hat{\imath}$ - $\hat{\jmath}$ - $\hat{k}$) + μ($2\hat{\imath}$ + $\hat{\jmath}$) and ($2\hat{\imath} - \hat{\jmath}$) + α($\hat{\imath}$ + $\hat{\jmath}$ - $\hat{k}$) intersect or not. If intersecting find their point of intersection.

19. Evaluate $\int_0^1 \sin^{-1}\left(x\sqrt{1-x} - \sqrt{x\sqrt{1-x^2}}\right) dx$
20. If the sum of the two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Or,

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}.\vec{b}$=$\vec{a}.\vec{c}$=0 and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6.}$ Prove that $\vec{a}=\pm 2(\vec{b}\text{x}\vec{c})$

21. Solve the following differential equation : (xdy-ydx)y.sin($\frac{y}{x}$) = (ydx-xdy)x.cos($\frac{y}{x}$) given that y=π when x=3
22. Show that the four points (0,-1,-1), (4,5,1), (3,9,4) and (-4,4,4) are coplanar. Also find the equation of the plane containing them.

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Section – C

(Questions 23 – 29 carry Six marks each)

23. Using the method of integration, find the area of the region {(x,y) : |x-1|≤y≤ $\sqrt{5-x^2}$

24. Find the angle between the line (x+1)/2 = y/3 = (z-3)/(6) and the plane 10x + 2y – 11z = 3

25. From a pack of 52 cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both diamonds. What is the probability that the lost card was a card of heart.

26. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertival angle α, is $\frac{4}{27}\pi h^3 tan^2\alpha$
27. Evaluate the following : $\int_{-1}^{\frac{1}{2}} |\ xcos\pi x\ |\ dx$
28. Evaluate $\int_0^{\frac{\pi}{2}} logsinx\ dx$
29. A diet of a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods A and B are available at a cost of Rs.5 and Rs.4 per unit respectively. One unit of food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of food B contains100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of foods A and B should be used to have least cost, but it must satisfy the requirements of the sick person. Form the equation as LPP and solve it graphically.

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