D.V.R'S INSTITUTE OF MATHEMATICS

CHENNAI – 600 050 (9940116934)

XII CBSE - MATHEMATICS MODEL PAPER

2004 – 2005 - SET - 1

SECTION – A (6 x 1 = 6 Marks)

1. Evaluate $\tan^{-1}\left(\frac{3+4x}{4-3x}\right)$l; x < 4/3.

2. On the set Q^{+} of all positive rational numbers a binary operation * is defined by $a * b = \frac{ab}{2}$ for all a,b $\in Q^{+}$, Find the identity element.

3. Verify for $\left(A^{-1}\right)^{-1} = A$, for $A = \begin{pmatrix} 2\sin\theta & \cos\theta \\ -2\cos\theta & \sin\theta \end{pmatrix}$

4. Find the projection of $7\vec{i} + \vec{j} - 4\vec{k}$ *on* $2\vec{i} + 6\vec{j} + 3\vec{k}$.

5. Write the value of $\hat{i}x(\hat{j}x\hat{k}) + \hat{j}x(\hat{k}x\hat{i}) + \hat{k}x(\hat{i}x\hat{j})$.

6. Find the perpendicular distance from the point (1,2,3) to the plane 3x+2y+z+10=0.

SECTION – B (13 x 4 = 42 Marks)

7. Evaluate $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$, if $\pi < x < \frac{3\pi}{2}$.

8. In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways, telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{telephone} \\ \text{House call} \\ \text{Letter} \end{matrix} \quad (\text{Cost per contact})$$

. The number of contact of each type made in two cities X and Y is given by

$$B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10{,}000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix} \quad (\text{Tele, House, Letter})$$

Find the total amount spent by the group in the two cities X and Y.

9. Using properties of determinants show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

10. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

11. Differentiate w.r.t.x $y = (\sin x)^{x} + (\cos x)^{\tan x} + \sin^{-1}\sqrt{x}$.

12. If $y = \left(\tan^{-1} x\right)^2$, Prove that $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$.

13. Evaluate $\int \frac{Cosx}{(1-Sinx)(2-Sinx)}\,dx$.

14. Find a and b so that $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{b(x)^{\frac{3}{2}}}, & x > 0 \end{cases}$ is continuous at x = 0.

15. Evaluate $\int x^2 \tan^{-1} x \, dx$

16. Evaluate $\int_0^{\frac{\pi}{2}} \left(\frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} \right) dx$

17. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}, \vec{b} = 2\vec{i} + \vec{k}, \vec{c} = 2\vec{i} + \vec{j} + \vec{k}$, and $\vec{d} = \vec{i} + \vec{j} + 2\vec{k}$ then find the magnitude and direction cosines of $(\vec{a} \bullet \vec{b})(\vec{c} x d)$

18. A box contains 12 bulbs of which 3 are defective. If 3 bulbs are drawn from the box at random, find the probability distribution of X, the number of defective bulbs drawn. Hence compute the mean of X.

19. Find the distance of the point with position vector $-\hat{i} - 5\hat{j} - 10\hat{k}$ from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 12\hat{k})$ with the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

SECTION – C (7 x 6 = 42 Marks)

20. Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{(\sqrt{y+6}) - 1}{3} \right)$. (OR) Let * be a binary operation defined on NXN, by (a,b)*(c,d) = (ac, bd).

Show that * is commutative and associative. Also find the identity element for * on NxN.

21. A page of a book must have 18 sq.cm. of printed matter and must have 2 cm margins at the top and bottom and 1 cm. margin on each side. What dimension of the page will require the least amount of paper?

22. Find the area lying above x-axis and included between the circle $x^2+y^2=8x$ and the parabola $y^2 = 4x$.

23. Solve $\frac{y}{x}\cos\frac{y}{x} - \left(\frac{x}{y}\sin\frac{y}{x} + \cos\frac{y}{x} \right) dy = 0$

24. Find the equation of the plane passing through the intersection of the planes 2x+3y–z+1 = 0 x+y–2z+3 = 0 and perpendicular to the plane 3x–y–2z–4 = 0. Also find the inclination of this plane with xy-plane.

25. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain

drugs reduces its chance by 25%. At a time a patient can choose any one of two options with equal probability It is given that after going through one of two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. What are the benefits of meditation and yoga?

26. A company manufactures, two types of toys-A and B. Toy A require 4 minutes for cutting and 8 minutes for assembling and Toy B requires 8 minutes for assembling. There are 3 hours and 20 minutes available in a day for cutting and 4 hours for assemble. The profit on a piece of toy A is Rs. 50 and that on toy B is Rs. 60. How many toys of each type should be made daily to have maximum profit? Solve the problem graphically.

D. Vasuraj, M.Sc., B.Ed.,
D.V.R'S Institute of Mathematics,
Chennai - 600 050.
vraj626@gmail.com
9940116934