CLASS XII
GUESS PAPER
MATHS

TIME: 3 hrs. DATE: __.__.__ MAX. MARKS: 70

INSTRUCTIONS:

- The question paper consists of 3 pages.
- Diagrams/graphs are to be drawn only with a PENCIL; Calculators are NOT allowed.
- The paper consists of 30 questions with following marks distribution:
    - Section A consists of 10 questions of 1 mark each.
    - Section B includes questions 11 to 22 of 4 marks each.
    - Section C includes questions 23 to 29 of 6 marks each.

SECTION A

Q.1: If f: [0,1] → [0,1], defined by f(x) = x^2 and g: [0,1] → [0,1], defined by g(x) = 1 – x, then determine f(g(x)).

Q.2: Show that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.

Q.3: If A = $\begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix}$. Find a skew symmetric matrix using A and A^T.

Q.4: Find adjoint of the matrix A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Q.5: Find the value of $\begin{vmatrix} 109 & 102 & 95 \\ 6 & 13 & 20 \\ 1 & -6 & -13 \end{vmatrix}$.

Q.6: Evaluate $\int 3\sqrt{x}\,(1+\sqrt{x^3})\,dx$.

Q.7: Evaluate $\int_0^{400\pi} \sqrt{1-\cos 2x}\;dx$.

Q.8: If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}) \cdot (\vec{x}+\vec{a}) = 8$. Find $|\vec{x}|$.

Q.9: Find the angle between the planes

$2x - 3y + 4z = 1 \;\; and - x + y = 4$.

Q.10: Find the domain of the function f(x) = log x^2 .

SECTION B

Q.11: Let X = Y = R – {1}. Show that the function f: X → Y defined by $f(x) = \frac{x+2}{x-1}$ is one-one and onto. Find $f^{-1}(x)$.

Q.12: If $(\omega) = \begin{vmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{vmatrix}$, then show that $F(\alpha)F(\beta) = F(\alpha + \beta)$.

OR

Using properties of determinants, prove that

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x).$$

Q.13: Show that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2} \;\; where \; -\frac{\pi}{2} < x < \frac{3\pi}{2}$.

Q.14: Find the domain and range of the following function:

$$f(x) = \frac{x^2+x+1}{x^2+4x+3}.$$

Q.15: If $x = t\, cost \; and \; y = t + sint$, then find $\frac{d^2x}{dy^2}$ at t = $\frac{\pi}{4}$.

OR

Let $f(x) = sinx, \; g(x) = x^2, \; h(x) = \log_e x$. If U(x) = h(f(g(x))), then prove that

$$\frac{d^2U}{dx^2} = 2cotx^2 - 4x^2cosec^2x^2.$$

Q.16: Show that the curves

$xy = a^2 \;\; and \; x^2 + y^2 = 2a^2$ touch each other.

OR

Find the equation of normal to the curve

$y = \frac{1+sinx}{cosx}$ at $x = \frac{\pi}{4}$.

Q.17: Evaluate $\int \frac{x^3+3x+2}{(x^2+1)^2\,(x+1)}\,dx.$

OR

Evaluate $\int_0^{\pi/2} \{2\log(sinx) - \log(sin2x)\}\,dx.$

Q.18: Solve the differential equation

$\frac{dy}{dx} + \frac{2}{x}y = \frac{1}{x^2}$ given $y(2) = 2y(-1).$

Q.19: Form the differential equation corresponding to the function $y^2 = a(b-x)(b+x).$

Q.20: Find the value of the constant M such that the scalar product of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with the unit vector parallel to the sum of the vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $M\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to one.

Q.21: (a) Find the length of the perpendicular from (1 , -1 , 2) to the plane 3x + 5y -4z = 5.

(b) Prove that the angle between the line $\frac{x-2}{-1} = \frac{y+1}{1} = \frac{z-1}{1}$ and the plane 3x + 2y – z = 4 is $\sin^{-1}(\frac{-2}{\sqrt{42}})$.

Q.22: If the papers of 4 students can be checked by any of the 7 teachers, then show that the probability that all the 4 papers are checked by exactly 2 teachers is $\frac{6}{49}$.

SECTION C

Q.23: Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations

$x-2y+2z=1,\ 2y-3z=1,\ 3x-2y+4z=2.$

OR

Find the inverse of the matrix $\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}$ using elementary transformation.

Q.24: A window is in the form of a rectangle surmounted by a semi circle. Total perimeter of the window is 10 metres. Find the dimensions of the rectangle so as to admit maximum light through the whole opening.

Q.25: Using the method of integration, find the area of the region bounded by the lines $2x + y = 4,\ 3x - 2y = 6,\ x - 3y + 5 = 0$

Q.26: Solve the differential equation $(xdy - ydx)\ y\ sin\left(\frac{y}{x}\right) = (ydx + xdy).x\ cos\left(\frac{y}{x}\right)$ given that $y = \pi$ when x = 3.

Q.27: Find the equation of the plane passing through the point (-1 , 3 , 2) and perpendicular to each of the planes $x + 2y + 3z = 5\ \ and\ \ 3x + 3y + z = 0$.

Q.28: Given three identical boxes A, B and C each containing two coins. In box A, both the coins are gold coins, in box B both are silver coins and in box C there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold.

Q.29: A factory makes badminton rackets and cricket bats. A racket takes 90 minutes of machine time and 3 hours of craft man's time while the bat takes 3 hours of machine time and 60 minutes of craft man's time. In factory, machine is not available for more than 42 hours and that of craft man for 24 hours. If the profit on a racket and a bat is Rs. 20 and Rs. 10 respectively, find the number of rackets and bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P. and solve graphically.