

# CLASS XII SAMPLE PAPER MATHS

## SECTION A

**(Questions 1 to 10 carry 1 mark each)**

1. Let \* be the binary operation on N given by  $a * b = \text{HCF of } a \text{ and } b$ . Find  $20 * 16$
2. What is  $\sin^{-1}(\sin 7\pi/6)$  ?
3. Find x and y if  $\begin{bmatrix} 2x + y & 3 \\ 4 & x + 2y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
4. If A is a square matrix of order 3 and  $|\text{adj } A| = 64$  then find  $|A|$ .
5. Find  $\int \frac{1 + \cos x}{x + \sin x} dx$ .
6. Find the adj A of  $\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$ .
7. Evaluate:  $\int_{\frac{\pi}{2}}^{\pi} x^{10} \sin^{-1} x dx$
8. Find the value of  $\alpha$  so that  $\vec{a} = \alpha \hat{i} + 2\hat{j} + k\hat{k}$  is perpendicular to  $\vec{b} = 4\hat{i} - 9\hat{j} + 2\hat{k}$
9. Find the unit vector in the direction of  $\vec{a} + \vec{b}$  if  $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$  and  $\vec{b} = \hat{i} - \hat{j} - 5\hat{k}$ .
10. Find k if the lines  $\frac{x-1}{2} = \frac{y+1}{3k} = \frac{z-4}{5}$  and  $\frac{x+1}{1} = \frac{1-y}{2} = \frac{z+1}{3}$  are perpendicular.

## SECTION B(Q. 11 to 22 carry 4 marks each)

11. Show that the relation R on  $\mathbb{N} \times \mathbb{N}$  defined by  $(a,b) R (c,d) \Leftrightarrow a+d = b+c$  is an equivalence relation. (or)  
Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by  $f(x) = 4 + 3x$ . Show that f is invertible and find the inverse of f.
12. Prove that  $\tan^{-1} \left( \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \pi/4 - \frac{1}{2} \cos^{-1} x$ .

13. Using properties of determinants Prove that 
$$\begin{vmatrix} -\alpha^2 & \alpha\beta & \alpha\gamma \\ \beta\alpha & -\beta^2 & \beta\gamma \\ \gamma\alpha & \beta\gamma & -\gamma^2 \end{vmatrix} = 4\alpha^2\beta^2\gamma^2.$$

14. Test the continuity of the following function at  $x = 0$ ,

$$f(x) = \begin{cases} \frac{5x + |x|}{3x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

If  $x = a(t + \sin t)$ ,  $y = a(1 - \cos t)$ , show that  $y'' = 1/a$ , at  $t = \frac{\pi}{2}$ . (or) If  $x^p y^q = (x + y)^{p+q}$ , Prove that  $y' = y/x$ .

15. Find the intervals where the function  $f(x) = 2x^3 - 9x^2 + 12x + 30$  is a) increasing b) decreasing.

16. Evaluate:  $\int \frac{x^4+2}{x^2-1} dx$

(or)

Evaluate as sum of limits  $\int_1^3 (x^2 - x) dx$

17. Solve the differential equation  $x^2 y' = x^2 - 2y^2 + xy$

(or)

Form the differential equation representing the family of ellipses having foci on x-axis and centre at the origin.

Solve the differential equation  $\cos^2 x y' + y = \tan x$ .

18. Three vectors  $\vec{a}, \vec{b}, \text{ and } \vec{c}$  satisfying the condition  $\vec{a} + \vec{b} + \vec{c} = 0$ . Evaluate the quantity  $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  if  $|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$ .

19. Find the shortest distance between the lines  $\vec{r} = \vec{i} + \vec{j} + k(2i - j + k)$  and  $\vec{r} = (2i + j - k) + p(3i - 5j + 2k)$ .

20. In a factory which manufactures bolts, machine A, B and C respectively 25%, 35% and 40% of the bolts, Of their output 5, 4, and 2 percent are respectively defective bolts. A bolt is drawn random from the product and is found to be defective. What is the probability that it is manufactured from machine A?

### SECTION C ( Each question carries 6 marks)

21. Find the inverse of  $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$  using elementary transformation. ( or ) if  $A =$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \text{ find } A^{-1} \text{ and hence solve the equations } 2x+3y+z=11, -3x+2y+z=4,$$

$$5x-4y-2z = -9$$

24 .Find the maximum area of the isosceles triangle inscribed in an ellipse  $x^2/a^2 + y^2/b^2 = 1$ , whose vertex lies along the major axis. (or) Show that the maximum value of the cylinder which can be inscribed in a sphere of radius  $5\sqrt{3}cm$  is  $500\pi cm^3$ .

25. Prove that  $\int_0^{\frac{\pi}{2}} \sin 2x \cdot \log \tan x \cdot dx$ .

26. Make a rough sketch of the region given below and find its area using integration.  $\{ (x,y) : 0 \leq y \leq 2x+3, 0 \leq y \leq x^2 + 3 \}$ .

27. Find the foot of the perpendicular and the perpendicular distance of the point (3,2,1) from the plane  $2x-y+z+1=0$ . Find the image of the point in the plane.

28. From a lot of 30 bulbs which includes 6 defective, a sample of 4 bulbs is drawn at random with replacement. Find the mean and variance of the number of defective bulbs.

29. A furniture firm manufactures chairs and tables each requiring the use of three machines A,B and C . Production of the chair requires 2 hrs on machine A, 1 hr on machine B, and 1 hr on machine C. Each table requires 1 hr on machine A, 1 hr on machine B and 3 hrs on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table Rs. 60. The total time available per week on machine A is 70 hrs, machine B 40 hrs, and on machine C 90 hrs. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as LPP and solve it graphically.