ARMY PUBLIC SCHOOL JAMMU CANTT
PRE BOARD EXAMINATION 2011-12

CLASS: XII **M.M : 100**

SUBJECT: MATHEMATICS **TIME: 3 h**

GENERAL INSTRUCTION:

(a) All questions are compulsory.
(b) This question paper consists of 29 questions divided into three section A, B, and C. Section A comprises of 10 question of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.
(c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(d) There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
(e) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION A

Q1: Discuss equivalence of the relation given by $R = \{(x,y): x \leq y^2 \ , \ x, y \in R\}$.

Q2: For the principal values, evaluate $\sin^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.

Q3: Integrate $\int log x \, dx$

Q4: Find the angle between the two vectors $\vec{a}$ and $\vec{b}$ with magnitude 2 and 1 respectively and $\vec{a}.\vec{b} = \sqrt{3}$.

Q5: If $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$, Evaluate $|3A|$

Q6: A line passes through the points A (6, – 7, –1) and B (2, –3, 1).Find the direction cosines of the line so directed that the angle $\propto$ is acute.

Q7: Evaluate $\int_0^{\frac{\pi}{2}} log\left[\frac{3+5cosx}{3+5sinx}\right] dx$.

Q8: Find a unit vector in the direction of $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$ of magnitude 6.

Q9: Find projection of $\vec{a} = (3\hat{\imath} - \hat{\jmath} + 2\hat{k})$ on $\vec{b} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

Q10: Find values of x and y if $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$

SECTION–B

Q11: Prove that : $\tan^{-1}\left(\frac{63}{16}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$.

Q12: If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

OR

Verify Rolle's theorem for $f(x) = \sin^4 x + \cos^4 x$ on $[0, \frac{\pi}{2}]$. Hence find the point where tangent is parallel to x- axis.

Q13: Let $f : N \to R$ be *a* function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to$ Range (f) is invertible. Also find f^{-1}.

Q14: Evaluate : $\int \frac{2x-3}{\sqrt{(x-1)(2x+3)}} dx$

Q15: Find the equation of the plane through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$.

Q16: Show that the points A, B, C with position vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ respectively are the vertices of a right angled triangle. Also find the remaining angles.

Q17: Evaluate : $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x} dx$

Q18: Without expanding show that $\begin{vmatrix} a^2+1 & ab & ac \\ bc & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

Q19: Find the intervals in which $f(x) = \sin x + \cos x$, where $0 < x < 2\pi$ is increasing or decreasing.

OR

Find the equations of normal and tangent to the curve x=a $\sin^3 t$, $y = a\cos^3 t$ at t=$\frac{\pi}{4}$

Q20: Solve the following differential equation : $\cos^2 x \frac{dy}{dx} + y = \tan x$

OR

Form the differential equation of family of hyperbolas having x- axis as transverse axis.

Q21. A, B in order toss a coin. The first one to throw a head wins. What are their respective chances of winning? Assume that the game may continue indefinitely.

OR

A man can hit the target 3 out of 4 times How many minimum number of times should he fire so that his probability of hitting the target atleast once is more than 0.99?

Q22. Show that the differential equation $2y\, e^{x/y}\, dx + (y - 2x\, e^{x/y})\, dy = 0$ is homogeneous and find its particular solution given that x = 0 when y =1

SECTION-C

Q23. Show that the volume of the greatest cylinder that can be inscribed in a cone of height h and semi-vertical angle $\propto$ *is* $\frac{4}{27}\pi h^3 \tan^2 \propto$.

Q24. A factory has two machines A and B .Past records shows that machine A produced 60% of the output and machine B produced 40% of the output. Further , 2% of the items produced by machine A were defective and 1% produced by machine B were defective . All the items are put in one stockpile and then one item is chosen at random which is found to be defective . What is the probability that it was produced by machine A.

Q25. A variable plane which remains at a constant distance 3p from the origin, cuts the coordinate axes at A , B and C . Show that the locus of the triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$

OR

Find the image of the point (1, 2, 3) in the line $\vec{r} = (6\hat{i} + 7\hat{j} + 7\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$

Q26. Use limit sum method to evaluate $\int_{-1}^{3}(2x^2 - 4x + 3)dx$.

Q27. Sketch the graph of y=$|x + 2|$ and evaluate the area under the curve y = $|x + 2|$ above x-axis and between x= -5 and x=1.

Q28. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D,E and F whose requirement are 4500L, 3000L and 3500L respectively. The distance (in km) between the depots and the petrol pumps are given in the following table:

Distance in (km.)		
From/To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Re 1 per Km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

Q29 Find A^{-1}, if $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & 4 \end{bmatrix}$, Use elementary transformations.

OR

Solve the following system of equations: $2x - y + 3z = 5$, $3x + 2y - z = 7$, $4x + 5y - 5z = 9$

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