School of Math

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Sample Paper From N.C.E.R.T. Book

Time : 3hr. **MM : 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper contains* ***29*** *questions.*
3. *Questions 1 – 4 in section A are very short – answer type questions carrying 1 marks each.*
4. *Questions 5-12 in Section B are short – answer type questions carrying 2 marks each.*
5. *Questions 13-23 in section C are long – answer – I type questions carrying 4 marks each.*
6. *Questions 24 – 29 in section D are long – answer – II type questions carrying 6 marks each.*

Section-A

Q1 Let S = {a,b,c} and T= { 1,2,3} . Find F^{-1} of the following functions F from S to T, if it exists. 1
F= {(a,3), (b,2), (c,1)}

Q2 Find value of x, if 1

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

Q3 Find the following integral: 1

$$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

Q4 Determine order and degree (if defined)of differential equation: 1

$$y'' + 2y' + \sin y = 0$$

Section – B

Q5 Let f: { 1,3,4} $\rightarrow${1,2,5} and g: {1,2,5} $\rightarrow${1,3} be given by $f = \{(1,2),(3,5),(4,1)\}$ and g = {(1,3), (2,3),(5,1)}. Write down gof. 2

Q6 Express $\tan^{-1} \frac{\cos x}{1-\sin x}, \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. 2

Q7 If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ *and* $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that A^2 = kA – 2 I . 2

Q8 By using properties of determinant, show that: 2

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

Q9 Find the values of k so that the function f is continuous at the indicated point 2

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, if & x \neq \frac{\pi}{2} \\ 3, \quad if & x = \frac{\pi}{2} \end{cases} at\ x = \frac{\pi}{2}$$

Q10 Evaluate $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$ 2

Q11 From the differential equation of the family of circles having centre on y – axis and radius 3 2 units.

Q12 One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. 2

Section – C

Q13 Show that if $f : \mathrm{R}-\left\{\frac{7}{5}\right\} \rightarrow R-\left\{\frac{3}{5}\right\}$ is defined by $f\left(x\right)=\frac{3x+4}{5x-7}$ and 4

g:R- $\left\{\frac{3}{5}\right\} \rightarrow R-\left\{\frac{7}{5}\right\}$is defined by $g\left(x\right)=\frac{7x+4}{5x-3}$, then fog = I_A and gof = I_B , where ,

$A=R-\left\{\frac{3}{5}\right\}, B=R-\left\{\frac{7}{5}\right\}; \mathrm{I}_A(x)x, \forall x \in A, I_B(x)=x, \forall x \in B$ are called identity functions on sets A and B , respectively.

Q14 Prove that : 4

$$\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

OR

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x \in\left(0,\frac{\pi}{4}\right)$$

Q15 Using elementary transformation, find the inverse of the matrices , if it exists 4

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Q16 Show that 4

$$\Delta=\begin{vmatrix} \left(y+z\right)^2 & xy & zx \\ xy & \left(x+z\right)^2 & yz \\ xz & yz & \left(x+y\right)^2 \end{vmatrix}=2xyz\left(x+y+z\right)^3$$

Q17 If $y=\left(\tan^{-1}x\right)^2, show\ that \left(x^2+1\right)^2 y_2+2x\left(x^2+1\right)y_1=2$ 4

OR

Differentiate the function

$$\left(x+\frac{1}{x}\right)^x+x^{\left(1+\frac{1}{x}\right)}$$

Q18 $Differentitate \sqrt{\frac{(x-3)\left(x^2+4\right)}{3x^2+4x+5}}$ w.r.t.x 4

Q19 Find intervals in which the function given by 4

$$f\left(x\right)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$$

is (a) strictly increasing (b) strictly decreasing.

Q20 Show that the function f given by 4

$f\left(x\right)=x^3-3x^2+4x, x \in R$ is strictly increasing on R.

Q21 Evaluate the following definite integral as limit of sums. 4

$\int_0^4 \left(x+e^{2x}\right)dx$

Q22 Evaluate $\int_{-1}^{\frac{3}{2}} \left|x\sin(\pi x)\right| dx$

OR

Find $\int \left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$

Q23 Show that the given differential equation is homogeneous and solve it. 4

$$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

Section- D

Q24 Solve the system of the following equation using matrix method 6

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

Q25 A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. 6

Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

OR

Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one – third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2\alpha$

Q26 Evaluate the definite integrals 6

$\int_1^4 \left[|x-1|+|x-2|+|x-3|\right]dx$

Q27 Find the area lying above x – axis and included between the circle $x^2+y^2=8x$ and inside of the parabola $y^2 = 4x$. 6

Q28 Show that the differential equation $(x-y)\frac{dy}{dx}=x+2y$ is homogeneous and solve it. 6

Q29 A toy company manufactures two types of dolls. A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. A further, the production of level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to 3maximize the profit? 6

OR

A manufacturer has three machines I,II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for atleast 5 hours a day. She produces only two items M and N each requiring the use of all the three machines.

The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M N	1 2	2 1	1 1.25

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximize her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

Ans :1 $F^{-1} = \{(3,a),(2,b),(1,c)\}$ 2 x = 2 3 $\frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$ 4 order 2 degree 1

5 gof = {(1,3),(3,1),(4,3)} 6 $\frac{\pi}{4} + \frac{x}{2}$ 7 k = 1 **9** k = 6 10 $\frac{\pi^2}{32}$ 12 maximum no. of cakes – 30 of kind one and 10 cakes of another kind. 17 $\left(x + \frac{1}{x}\right)^x + \left[\frac{x^2 - 1}{x^2 + 1} + \log\left(x + \frac{1}{x}\right)\right] + x^{1+\frac{1}{x}}\left(\frac{x + 1 - \log x}{x^2}\right)$

18 $\frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left[\frac{1}{(x-3)} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$

19 f is strictly increasing in the interval $(3,\infty)$. 21 $\frac{15+e^8}{2}$ 22 $\frac{3}{\pi} + \frac{1}{\pi^2}$ or $x\log(\log x) - \frac{x}{\log x} + C$

24 x = 2, y = 3, z = 5 26 $\frac{19}{2}$ 27 $\frac{4}{3}(8+3\pi)$ 29 800 DOLLS of type A and 400 dolls of type B; maximum profit = Rs 16000 or 4000