CLASS – X (EXPECTED QUESTIONS) MATHS

Timeallowed: 3hrs **(2017–18)** **Marks: 80**

General Instructions:

(i) All questions are compulsory.

(ii) *The question paper consists of* ***30*** *questions divided into four sections* ***A, B, C*** *and* ***D.***

(iii)*Section* ***A*** *contains* ***6*** *questions of* ***1*** *mark each. Section* ***B*** *contains* ***6*** *questions of* ***2*** *marks each. Section* ***C*** *contains* ***10*** *questions of* ***3*** *marks each. Section* ***D*** *contains* ***8*** *questions of* ***4*** *marks each.*

(iv) *There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.*

(v) *Use of* ***calculators*** *is not permitted.*

Section-A
(Q.1 to 6 carry 1 mark each)

1. Can two numbers have 18 as their HCF and 380 as their LCM? Give reason.
2. Find the value(s) of k, if the quadratic eqn. $3x^2 - k\sqrt{3}\,x + 4 = 0$ has equal roots.
3. If the sum of the first q terms of an A.P. is $2q + 3q^2$, what is its common difference?
4. What is the distance between the points A(c, 0) and B(0, $-c$)?
5. If $\sec^2\theta\,(1 + \sin\theta)\,(1 - \sin\theta) = k$, then find the value of k.
6. In ΔLMN, ∠L = 50° and ∠N = 60°. If ΔLMN ~ ΔPQR, then find ∠Q.

Section-B
(Q.7 to 12 carry 2 mark each)

7. If two positive integers **p** and **q** are written as **p= a^{2}b^3** and **q= a^{3}b, a, b** are prime numbers, then verify: **L.C.M (p q)x H.C.F (pq) = pq**
8. Find the number of solutions of the following pair of linear equations:
   $x + 2y - 8 = 0$ $\quad\quad$ $2x + 4y = 16$
9. How many two-digit numbers are divisible by 3?
10. Find the relation between 'x' and 'y', if the points (x , y), (1,2) and (7,0) are collinear.
11. A box contains 5 red balls, 4 green balls, and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is
    (a) White $\quad\quad$ (b) Neither red nor white

12. A cone of height 20 cm and radius of base 5 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere.

Section-C
(Q.13 to 22 carry 3 mark each)

13. Show that exactly one of the numbers n, n+2, or n+4 is divisible by 3.

14. Find all the zeroes of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeroes are $-\sqrt{2}$ and $\sqrt{2}$.

15. If the sum of two natural numbers is 8 and their product is 15, find the numbers.

16. If the points A (4, 3) and B (x, 5) are on the circle with the centre O (2, 3), find the value of x.

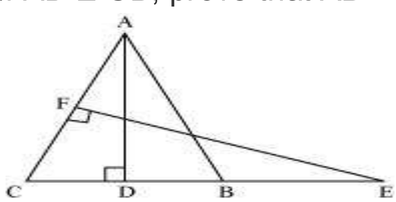
or

Prove that $3+\sqrt{2}$ is an irrational number

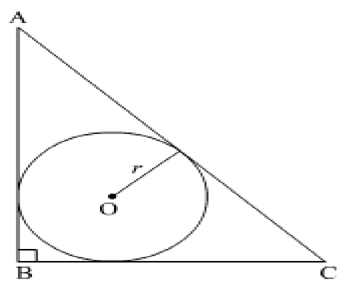
17. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

or

In figure , ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, Such that FE ⊥ AC. If AD ⊥ CB, prove that AB × EF = AD × EC.



18. In Figure a right triangle ABC, circumscribes a circle of radius r. If AB and BC are of lengths of 8 cm and 6 cm respectively, find the value of r.

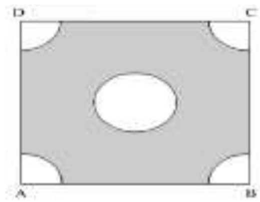


19. If $cos\theta - sin\theta = \sqrt{2}\, sin\theta$, prove that $cos\theta + sin\theta = \sqrt{2}\, sin\theta$

Or

Prove that $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

20. In Figure ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Find the area of the shaded region.
(Use π = 3.14)



21. A hemispherical tank, full of water, is emptied by a pipe at the rate of $\frac{25}{7}$ litres per sec. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m?

Or

Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

22. Find the mean of the following distribution:

Class	Frequency
0 – 10	8
10 – 20	12
20 – 30	10
30 – 40	11
40 – 50	9

Section-D
(Q.23 to 30 carry 4 marks each)

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23. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.

or

Solve the following for *x*:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

24. Find the common difference of an A. P. whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.

25. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Or

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.

26. Draw a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle.

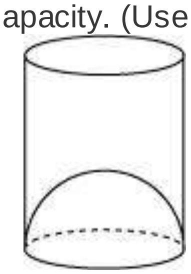
27. Prove the following:
$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

OR

Prove the following:
$\sin A (1 + \tan A) + \cos A (1 + \cot A) = \sec A + \operatorname{cosec} A$

28. The angles of elevation and depression of the top and bottom of a light-house from the top of a 60 m high building are 30° and 60° respectively. Find
(i) the difference between the heights of the light-house and the building.
(ii) the distance between the light-house and the building.

29. A juice seller serves his customers using a glass as shown in Figure 6. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14)



30. A Group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest.
Which of the above values you prefer more?

Or

During the medical check-up of 35 students of a class their weights were recorded as follows:

Weight (in kg)	Number of students
38 – 40	3
40 – 42	2
42 – 44	4
44 – 46	5
46 – 48	14
48 – 50	4
50 – 52	3

Draw a less than type and a more than type ogive from the given data. Hence obtain the median weight from the graph.

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Section-A
(Q.1 to 6 carry 1 mark each)

1. Can two numbers have 18 as their HCF and 380 as their LCM? Give reason.
   Ans: No. Because HCF is always a factor of LCM but here 18 is not a factor of 380.
2. Find the value(s) of k, if the quadratic eqn. $3x^2 - k\sqrt{3}\,x + 4 = 0$ has equal roots.
   Ans: k= =$\pm$ 4
3. If the sum of the first q terms of an A.P. is $2q + 3q^2$, what is its common difference?

Solution: Let a and d respectively be the first term and common difference of the A.P.

It is given that $S_q = 2q + 3q^2$
$\therefore S_1 = 2 \times 1 + 3 \times 1^2 = 5$
$\Rightarrow a_1 + S_1 = 5$
$S_2 = 2 \times 2 + 3 \times 2^2 = 4 + 12 = 16$
$\Rightarrow a_1 + a_2 = 16$
$\Rightarrow a + (a + d) = 16$
$\Rightarrow 2 \times 5 + d = 16$
$\Rightarrow d = 16 - 10 = 6$
Thus, the common difference of the A.P. is 6.

4. What is the distance between the points A(c, 0) and B(0, −c)?

Solution:

Using distance formula, the distance between the points A(c, 0) and B (0, −c) is given by:

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-c)^2 + (-c-0)^2}$ units

$= \sqrt{c^2 + c^2}$ units

$= \sqrt{2c^2}$ units

$= c\sqrt{2}$ units

Thus, the distance between the given is $c\sqrt{2}$ units**.**

5. If $\sec^2\theta\,(1 + \sin\theta)\,(1 - \sin\theta) = k$, then find the value of k.

Solution:

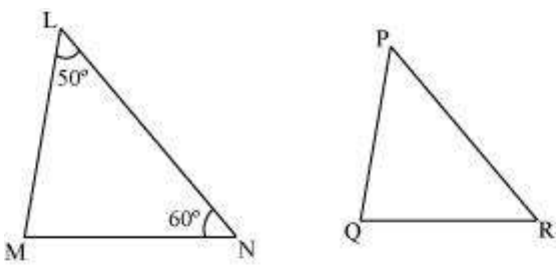
$\sec^2\theta\,(1 + \sin\theta)(1 - \sin\theta) = k$

$$\Rightarrow \sec^2\theta(1-\sin^2\theta)=k \qquad [(a+b)(a-b)=a^2-b^2]$$
$$\Rightarrow \sec^2\theta(\cos^2\theta)=k \qquad [1-\sin^2\theta=\cos^2\theta]$$
$$\Rightarrow \frac{1}{\cos^2\theta}\times\cos^2\theta=k \qquad \left[\sec\theta=\frac{1}{\cos\theta}\right]$$
$$\Rightarrow 1=k$$
$$\Rightarrow k=1$$

Thus, the value of k is 1.

6. In ΔLMN, ∠L = 50° and ∠N = 60°. If ΔLMN ~ ΔPQR, then find ∠Q.

Solution:



∠L + ∠M + ∠N = 180° (Angle sum property)
Substituting ∠L = 50° and ∠N = 60° in this equation:
50° + ∠M + 60° = 180°
∠M = 70°
It is given that ΔLMN ~ ΔPQR.
We know that corresponding angles in similar triangles are of equal measures.
∴ ∠M = ∠Q = 70°
Thus, the measure of ∠Q is 70°.

Section-B
(Q.7 to 12 carry 2 mark each)

7. If two positive integers **p** and **q** are written as **$p = a^2b^3$** and **$q = a^3b$**, **a, b** are prime numbers, then verify: **L.C.M (p q)x H.C.F (pq) = pq**

Ans: LCM (p, q) = a^3b^3
HCF (p, q) = a^2b

LCM (p, q) × HCF (p, q) = $a^5b^4 = (a^2b^3)(a^3b) = pq$

8. Find the number of solutions of the following pair of linear equations:
$x + 2y - 8 = 0 \qquad 2x + 4y = 16$

Solution:

The given pair of linear equations is
$x + 2y - 8 = 0$
$2x + 4y - 16 = 0$
On comparing with general equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

we obtain

$$\frac{a_1}{a_2} = \frac{1}{2},\ \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2},\ \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

Hence, the given pair of linear equations has infinitely many solutions.

9. How many two-digit numbers are divisible by 3?

Solution:

Two digit numbers, divisible by 3, are 12, 15, 18, …, 99.
The sequence of numbers 12, 15, 18, …, 99 is an A.P.
Here, first term (a) = 12 and Common difference (d) = 15 −12 = 3
Let 99 be the n^{th} term.
$\Rightarrow a_n = 99$
$\Rightarrow a + (n - 1)d = 99$
$\Rightarrow 12 + (n - 1)3 = 99$
$\Rightarrow 12 + 3n - 3 = 99$
$\Rightarrow 3n + 9 = 99$
$\Rightarrow 3n = 99 - 9$
$\Rightarrow 3n = 90$

$$\Rightarrow n = \frac{90}{3}$$
$$\Rightarrow n = 30$$

Thus, there are 30 two-digit numbers which are divisible by 3.

10. Find the relation between ‘x’ and ‘y’, if the points (x , y), (1,2) and (7,0) are collinear.

Soln: $\frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right.$

$\frac{1}{2}$ **x (2-0) +1(0-y) +7(y-2)=0**

X= 7-3y

11. A box contains 5 red balls, 4 green balls, and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is
(a) White (b) Neither red nor white

Solution:
The box contains 5 red balls, 4 green balls, and 7 white balls.
Therefore, total number of balls in the box = 5 + 4 + 7 = 16

(a) Probability that the ball drawn is white = $\frac{\text{Number of white balls}}{\text{Total number of balls in the box}} = \frac{7}{16}$

(b) Probability that the ball drawn is neither red nor white

$$= \frac{\text{Number of balls which are neither red nor white}}{\text{Total number of balls in the box}}$$
$$= \frac{\text{Number of green balls}}{\text{Total number of balls in the box}}$$
$$= \frac{4}{16}$$
$$= \frac{1}{4}$$

12. A cone of height 20 cm and radius of base 5 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere.

Soln. Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi \times 5^2 \times 20$$

r = 5 cm

Section-C
(Q.13 to 22 carry 3 mark each)

13. Show that exactly one of the numbers n, n+2, or n+4 is divisible by 3.

Ans. Let n = 3k, 3k + 1 or 3k + 2.
(i) When n = 3k:
n is divisible by 3.
n + 2 = 3k + 2 => n + 2 is not divisible by 3.
n + 4 = 3k + 4 = 3(k + 1) + 1 => n + 4 is not divisible by 3.

(ii) When n = 3k + 1:
n is not divisible by 3.
n + 2 = (3k + 1) + 2 = 3k + 3 = 3(k + 1) => n + 2 is divisible by 3.

n + 4 = (3k + 1) + 4 = 3k + 5 = 3(k + 1) + 2 => n + 4 is not divisible by 3.
(iii) When n = 3k + 2:
n is not divisible by 3.
n + 2 = (3k + 2) + 2 = 3k + 4 = 3(k + 1) + 1 => n + 2 is not divisible by 3.
n + 4 = (3k + 2) + 4 = 3k + 6 = 3(k + 2) => n + 4 is divisible by 3.
Hence exactly one of the numbers n, n + 2 or n + 4 is divisible by 3.

14. Find all the zeroes of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeroes are $-\sqrt{2}$ and $\sqrt{2}$.

Solution: The given polynomial is $p(x) = x^3 + 3x^2 - 2x - 6$.

It is given that $-\sqrt{2}$ and $\sqrt{2}$ are the two zeroes of $p(x)$.

Thus, $(x+\sqrt{2})$ and $(x-\sqrt{2})$ are the factors of $p(x)$.

This means, $(x+\sqrt{2})(x-\sqrt{2}) = x^2 - 2$ is also a factor of $p(x)$.

We can divide $p(x) = x^3 + 3x^2 - 2x - 6$ by $x^2 - 2$ as

$$
\begin{array}{r|l}
 & x+3 \\
\hline
x^2-2 & x^3 + 3x^2 - 2x - 6 \\
 & x^3 \quad\quad - 2x \\
 & - \quad\quad\quad + \\
\hline
 & 3x^2 \quad - 6 \\
 & 3x^2 \quad - 6 \\
 & - \quad\quad + \\
\hline
 & 0
\end{array}
$$

$\therefore p(x) = (x^2 - 2)(x + 3)$
As $(x + 3)$ is a factor of the polynomial $p(x)$, so $x = -3$ is a zero of the polynomial.
Thus, −3 is the third zero of the given polynomial.
Thus, the three zeroes of $p(x) = x^3 + 3x^2 - 2x - 6$ are −3, $-\sqrt{2}$, and $\sqrt{2}$.

15. If the sum of two natural numbers is 8 and their product is 15, find the numbers..

Soln: Let the two natural numbers be *a* and *b*.
It is given that, sum of two numbers = 8
$\therefore a + b = 8$
$\Rightarrow a = 8 - b$ ….(1)
It is given that, product of two numbers = 15
$\therefore ab = 15$

$\Rightarrow (8 - b)b = 15$ [Using (1)]
$\Rightarrow 8b - b^2 = 15$
$\Rightarrow b^2 - 8b + 15 = 0$
$\Rightarrow b^2 - 5b - 3b + 15 = 0$
$\Rightarrow b(b - 5) - 3(b - 5) = 0$
$\Rightarrow (b - 3)(b - 5) = 0$
$\Rightarrow b = 3$ 0r $b = 5$
When $b = 3$, we have
$a = 8 - b = 8 - 3 = 5$
When b = 5, we have
$a = 8 - b = 8 - 5 = 3$
Thus, the required natural numbers are 3 and 5.

16. If the points A (4, 3) and B (x, 5) are on the circle with the centre O (2, 3), find the value of x.

or

Prove that $3+\sqrt{2}$ is an irrational number

Solution: If the points A (4, 3) and B (x, 5) are on the circle with the centre O (2, 3), then OA and OB will be the radii of the circle.

$\therefore$OA = OB
$\Rightarrow OA^2 = OB^2$
$\Rightarrow (4 - 2)^2 + (3 - 3)^2 = (x - 2)^2 + (5 - 3)^2$ [Using distance formula]
$\Rightarrow 2^2 + 0^2 = (x - 2)^2 + 2^2$
$\Rightarrow 0 = (x - 2)^2$
$\Rightarrow x - 2 = 0$
$\Rightarrow x = 2$
Thus, the required value of x is 2.

- **or**

Prove that $3+\sqrt{2}$ is an irrational number.

Solution:

If possible, suppose $3+\sqrt{2}$ is rational.
Therefore, we can find two integers a, b ($b \neq 0$) such that

$$3+\sqrt{2}=\frac{a}{b}$$

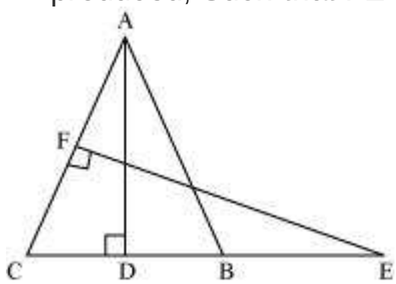
$$\Rightarrow \sqrt{2}=\frac{a}{b}-3$$

Since *a*, *b*, and 3 are integers, $\frac{a}{b}-3$ is a rational number. Hence, $\sqrt{2}$ should be rational.
This conclusion contradicts the fact that $\sqrt{2}$ is irrational.
Therefore, our assumption is false.
Hence, $3+\sqrt{2}$ is irrational.

17. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

or

In figure , ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, Such that FE ⊥ AC. If AD ⊥ CB, prove that AB × EF = AD × EC.

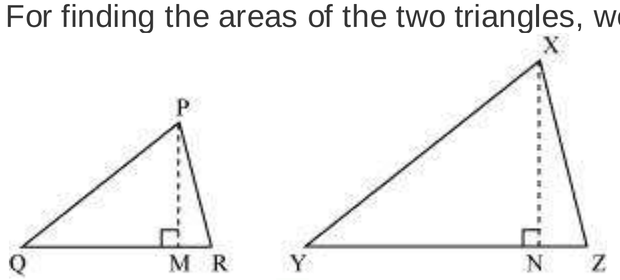


Solution:

We are given two triangles PQR and XYZ such that ΔPQR ~ ΔXYZ

We have to prove that $\frac{ar(\Delta PQR)}{ar(\Delta XYZ)}=\left(\frac{PQ}{XY}\right)^2=\left(\frac{QR}{YZ}\right)^2=\left(\frac{RP}{ZX}\right)^2$

For finding the areas of the two triangles, we draw PM ⊥ QR and XN ⊥ YZ.



Now, ar (ΔPQR) = $\frac{1}{2}$ QR × PM

ar (ΔXYZ) = $\frac{1}{2}$ YZ × XN

$$\therefore \frac{ar(\Delta PQR)}{ar(\Delta XYZ)} = \frac{\frac{1}{2}QR \times PM}{\frac{1}{2}YZ \times XN} = \frac{QR}{YZ} \times \frac{PM}{XN} \quad ...(1)$$

Now, in ΔPQM and ΔXYN,
∠Q = ∠Y (∵ ΔPQR ~ ΔXYZ)
∠PMQ = ∠XNY (Each is equal to 90°)
∴ΔPQM ~ ΔXYN (AA similarity criterion)

$$\text{Thus, } \frac{PQ}{XY} = \frac{PM}{XN} \quad ...(2)$$

Also, since ΔPQR ~ ΔXYZ,

$$\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{RP}{ZX} \quad ...(3)$$

From equation (1), we obtain

$$\frac{ar(\Delta PQR)}{ar(\Delta XYZ)} = \frac{QR}{YZ} \times \frac{PM}{XN}$$
$$= \frac{PQ}{XY} \times \frac{PQ}{XY} \quad \text{[Using equations (2) and (3)]}$$
$$= \left(\frac{PQ}{XY}\right)^2$$

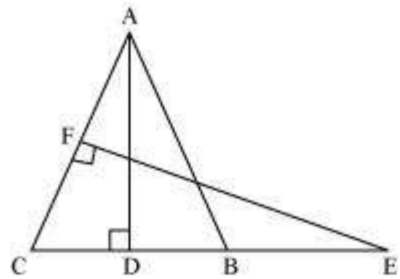
Now, by using equation (3), we obtain

$$\frac{ar(\Delta PQR)}{ar(\Delta XYZ)} = \left(\frac{PQ}{XY}\right)^2 = \left(\frac{QR}{YZ}\right)^2 = \left(\frac{RP}{ZX}\right)^2$$

Hence, the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Or

In figure ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, Such that FE ⊥ AC. If AD ⊥ CB, prove that AB × EF = AD × EC.



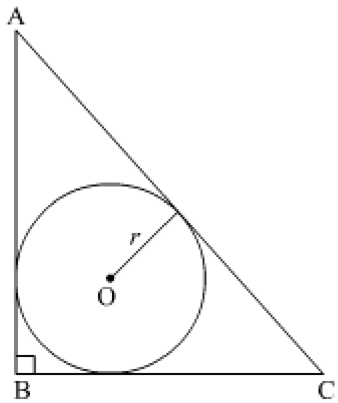
Solution:

It is given that the length of AB is same as that of AC.
It is known that angles opposite to equal sides of a triangle are equal.
$\therefore \angle ACB = \angle ABC$
$\Rightarrow \angle FCE = \angle ABD$ … (*i*)
In ΔABD and ΔEFC:
$\angle ADB = \angle EFC = 90^\circ$
$\angle ABD = \angle ECF$ {using (i) }
ΔADB ~ ΔEFC (By AA similarity criterion)
It is known that the corresponding sides of similar triangles are in proportion.

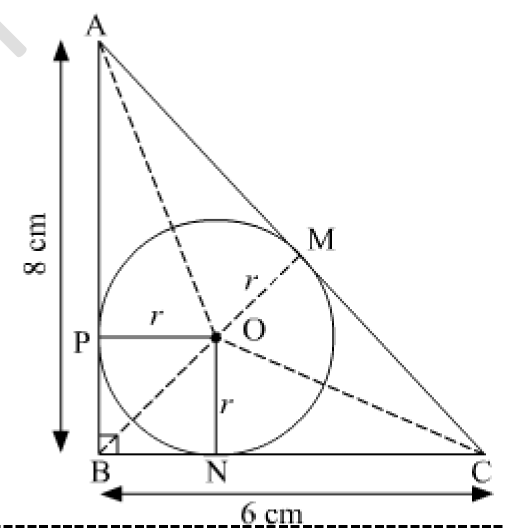
$$\therefore \frac{AB}{EC} = \frac{AD}{EF}$$

$$\Rightarrow AB \times EF = AD \times EC$$

18. In Figure a right triangle ABC, circumscribes a circle of radius *r*. If AB and BC are of lengths of 8 cm and 6 cm respectively, find the value of *r*.



Solution: Let ABC be the right angled triangle such that ∠B = 90° , BC = 6 cm, AB = 8 cm. Let O be the centre and *r* be the radius of the incircle.



AB, BC and CA are tangents to the circle at P, N and M.

∴ OP = ON = OM = *r* (radius of the circle)

$$\text{Area of } \Delta ABC = \frac{1}{2}\times 6\times 8 = 24 \text{ cm}^2$$

By Pythagoras theorem,

$CA^2 = AB^2 + BC^2$

$\Rightarrow CA^2 = 8^2 + 6^2$

$\Rightarrow CA^2 = 100$

$\Rightarrow CA = 10$ cm

Area of △ABC = Area △OAB + Area △OBC + Area △OCA

$$24 = \tfrac{1}{2}r\times AB + \tfrac{1}{2}r\times BC + \tfrac{1}{2}r\times CA$$

$$24 = \tfrac{1}{2}r\left(AB + BC + CA\right)$$

$$\Rightarrow r = \frac{2\times 24}{(AB + BC + CA)}$$

$$\Rightarrow r = \frac{48}{8+6+10}$$

$$\Rightarrow r = \frac{48}{24}$$

$$\Rightarrow r = 2 \text{ cm}$$

19. If $cos\theta - sin\theta = \sqrt{2}\, sin\theta,\ prove\ that\ cos\theta + sin\theta = \sqrt{2}\, sin\theta$

Or

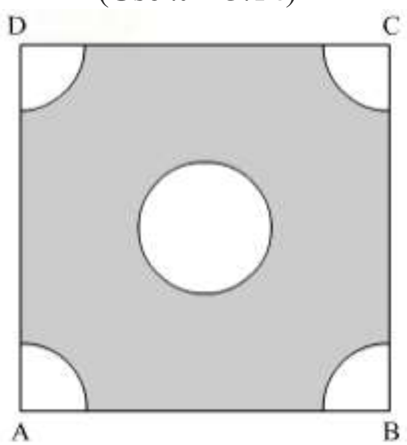
- Prove that $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1 + \sin A}{\cos A}$

Solution:

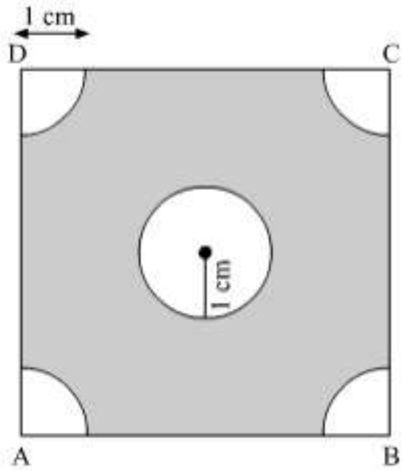
$$\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{\tan A+\sec A-\left(\sec^2 A-\tan^2 A\right)}{\tan A-\sec A+1} \qquad \left[\because 1+\tan^2 A=\sec^2 A\right]$$
$$=\frac{\tan A+\sec A-(\sec A-\tan A)(\sec A+\tan A)}{\tan A-\sec A+1}$$
$$=\frac{(\sec A+\tan A)(1-\sec A+\tan A)}{\tan A-\sec A+1}$$
$$=\sec A+\tan A$$
$$=\frac{1}{\cos A}+\frac{\sin A}{\cos A}$$
$$=\frac{1+\sin A}{\cos A}$$

.

20. In Figure ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Find the area of the shaded region.
(Use $\pi = 3.14$)



Solution:



ABCD is a square. A quadrant of circle of radius 1 cm is drawn at each vertex of the square.
The quadrant of a circle is a sector of angle 90°.

$\therefore$ Area of each quadrant $= \frac{90°}{360°} \pi r^2 = \frac{1}{4} \times 3.14 \times (1 \text{ cm})^2 = 0.785 \text{ cm}^2$

Area of square = $(\text{Side})^2 = (4 \text{ cm})^2 = 16 \text{ cm}^2$
Diameter of the circle = 2 cm
$\therefore$ Radius of the circle = 1 cm
$\therefore$ Area of circle = $\pi r^2 = 3.14 \times (1 \text{ cm})^2 = 3.14 \text{ cm}^2$
Area of shaded region
= Area of square − Area of circle − (4 × Area of each quadrant)
= $16 \text{ cm}^2 - 3.14 \text{ cm}^2 - (4 \times 0.785 \text{ cm}^2)$
= $16 \text{ cm}^2 - 3.14 \text{ cm}^2 - 3.14 \text{ cm}^2$
= $16 \text{ cm}^2 - 6.28 \text{ cm}^2$
= 9.72 cm^2
Thus, the area of the shaded region is 9.72 cm^2.

21. A hemispherical tank, full of water, is emptied by a pipe at the rate of $\frac{25}{7}$ litres per sec. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m?

Or

Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Solution:

It is given that, diameter of base of tank = 3 m

$\therefore$ Radius, $r = \frac{3}{2}$ m

Volume of water in the hemispherical tank

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\text{m}\right)^3$

$= \frac{99}{14}\text{m}^3$

Rate of flow of water out of the pipe = $\frac{25}{7}$ litres / sec
Let the time taken to empty half the tank be *t* sec.

∴ Rate of flow of water × *t* sec $= \frac{1}{2} \times$ Volume of water in the hemispherical tank

$\Rightarrow \frac{25}{7} \times t \text{ litre} = \frac{1}{2} \times \frac{99}{14}\text{m}^3$

$\Rightarrow \frac{25}{7} \times \frac{1}{1000} \times t\text{ m}^3 = \frac{1}{2} \times \frac{99}{14}\text{m}^3 \qquad \left(\because 1\text{ litre} = \frac{1}{1000}\text{m}^3\right)$

$\Rightarrow t = 990$

∴ Time taken to empty half the tank is (960 + 30) sec = 16 min 30 sec

Or

Ans. Let the area that can be irrigated in 30 minute be A m^2
Water flowing in canal in 30 minutes $= 10{,}000 \times \frac{1}{2} = 5000m$
Volume of water flowing out in 30 minutes $= 5000 \times 6 \times 1.5\, m^3 = 45000 m^3$-------(i)
Volume of water required to irrigate the field$= A \times \frac{8}{100}\, m^3$--------------(ii)
Equating (i) and (ii), we get $A \times \frac{8}{100} = 45000 = 562500\text{ m}^2$

22. Find the mean of the following distribution:

Class	Frequency
0 – 10	8
10 – 20	12
20 – 30	10
30 – 40	11
40 – 50	9

Solution:

We may find the class mark, x_i, of each interval by using the relation:

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size of this data, h = 10

Now, taking 25 as assumed mean (a), we may calculate d_i, u_i, f_iu_i as follows:

Class	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - 25$	$u_i = \frac{x_i - 25}{h}$	f_iu_i
0 − 10	8	5	− 20	− 2	− 16
10 − 20	12	15	− 10	− 1	− 12
20 − 30	10	25	0	0	0
30 − 40	11	35	10	1	11
40 − 50	9	45	20	2	18
Total	$\sum f_i = 50$	-	-	-	$\sum f_iu_i = 1$

From the table, it can be observed that:

$$\sum f_i = 50 \text{ and } \sum f_iu_i = 1$$

$$\therefore \text{Mean} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$$

$$= 25 + \left(\frac{1}{50}\right) \times 10$$

$$= 25 + 0.2 = 25.2$$

Section-D

(Q.23 to 30 carry 4 marks each)

23. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.

or

Solve the following for x:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

Solution: Let the original speed of the plane be x km/hr.
If the speed of the plane is reduced by 100 km/hr, then
Reduced speed of the plane = $(x - 100)$ km/hr

Time taken by the plane to reach its destination at original speed, $t_1 = \frac{2800}{x}$ hr

Time taken by the plane to reach its destination at reduced speed, $t_2 = \frac{2800}{x-100}$ hr
Given,
Time taken by the plane to reach its destination at reduced speed − Time taken by the plane to reach its destination at original speed = 30 minutes

$$\therefore t_2 - t_1 = \frac{1}{2} \text{ hr}$$
$$\Rightarrow \frac{2800}{x-100} - \frac{2800}{x} = \frac{1}{2}$$
$$\Rightarrow \frac{2800x - 2800(x-100)}{x(x-100)} = \frac{1}{2}$$
$$\Rightarrow 2800 \times 100 \times 2 = x^2 - 100x$$
$$\Rightarrow x^2 - 100x - 560000 = 0$$
$$\Rightarrow x^2 - 800x + 700x - 560000 = 0$$
$$\Rightarrow x(x-800) + 700(x-800) = 0$$
$$\Rightarrow (x-800)(x+700) = 0$$
$$\Rightarrow x - 800 = 0 \text{ or } x + 700 = 0$$
$$\Rightarrow x = 800 \text{ or } x = -700$$
$$\therefore x = 800 \qquad [\because \text{Speed cannot be negative}]$$

Original speed of flight = 800 km/hr

∴ Original duration of flight, $t_1 = \frac{2800}{800} = \frac{7}{2} = 3\frac{1}{2}$ Hrs
Thus, the original duration of the flight is 3 hours 30 minutes

or

The given equation is $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$.

$$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$
$$\Rightarrow \frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$$
$$\Rightarrow \frac{2x-2a-b-2x}{2x(2a+b+2x)}=\frac{b+2a}{2ab}$$
$$\Rightarrow \frac{-2a-b}{2x(2a+b+2x)}=\frac{b+2a}{2ab}$$
$$\Rightarrow \frac{-(2a+b)}{2x(2a+b+2x)}=\frac{b+2a}{2ab}$$
$$\Rightarrow \frac{-1}{x(2a+b+2x)}=\frac{1}{ab}$$
$$\Rightarrow 2x^2+2ax+bx+ab=0$$
$$\Rightarrow 2x(x+a)+b(x+a)=0$$
$$\Rightarrow (x+a)(2x+b)=0$$
$$\Rightarrow x+a=0 \text{ or } 2x+b=0$$
$$\Rightarrow x=-a, \text{ or } x=\frac{-b}{2}$$

24. Find the common difference of an A. P. whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.

Solution:

Let the common difference of the given A. P. be d.
First term (a) = 5 (Given)
We know that sum of the first n terms of an A.P. is given as:

$$S_n=\frac{n}{2}[2a+(n-1)d]$$
$$\therefore \text{Sum of first four terms } (S_4)=\frac{4}{2}[2\times5+(4-1)d]$$
$$=2[10+3d]$$
$$=20+6d$$

And, sum of next four terms = $S_8 - S_4$

$$=\frac{8}{2}[2\times5+(8-1)d]-(20+6d)$$
$$=40+28d-20-6d$$
$$=20+22d$$

According to the given condition,

$$S_4 = \frac{1}{2}[S_8 - S_4]$$
$$\Rightarrow 20 + 6d = \frac{1}{2}[20 + 22d]$$
$$\Rightarrow 20 + 6d = 10 + 11d$$
$$\Rightarrow 11d - 6d = 20 - 10$$
$$\Rightarrow 5d = 10$$
$$\Rightarrow d = \frac{10}{5}$$
$$\Rightarrow d = 2$$

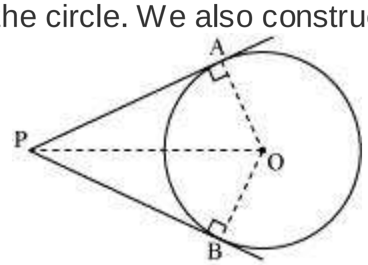
Hence, the common difference of the given A.P. is 2.

25. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Or

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.

Solution: The figure shows a circle with centre O. P is a point taken in the exterior of the circle. PA and PB are tangents from point P to the circle. We also construct OA, OB, and OP.



We need to prove that the lengths of the tangents drawn from an external point to a circle are equal, i.e., PA = PB.

It is known that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ PA and PB are perpendicular to OA and OB respectively.

⇒ ∠OAP = ∠OBP = 90°

In ΔOAP and ΔOBP:

OA = OB (Radii of the same circle)

OP = OP (Common)

∠OAP = ∠OBP (Each 90°)

∴ ΔOAP ≅ ΔOBP (RHS congruence criterion)

∴ PA = PB (CPCT)

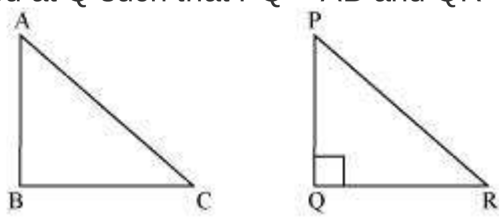
Thus, the lengths of the tangents drawn from an external point to a circle are equal.

Hence, proved.

Or

Solution:

Suppose we are given ABC in which $AC^2 = AB^2 + BC^2$. We have to prove that ∠90º
Let us construct PQR right-angled at Q such that PQ = AB and QR = BC



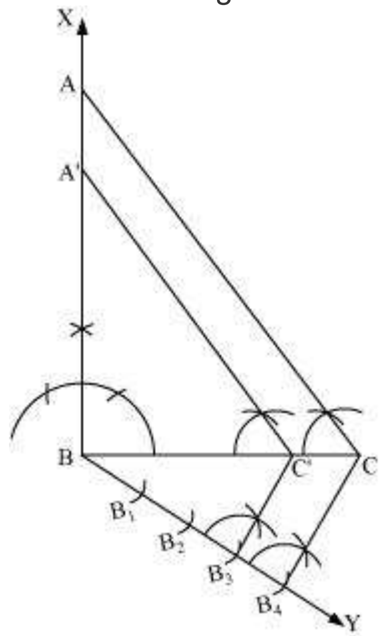
Applying Pythagoras theorem in PQR:
$PR^2 = PQ^2 + QR^2$
⇒ $PR^2 = AB^2 + BC^2$ …(i) (By construction)
However, $AC^2 = AB^2 + BC^2$ …(2) (Given)
From (1) and (2), we obtain:
AC = PR … (3)
Now, in ABC and PQR, we obtain:
AB = PQ (By construction)
BC = QR (By construction)
AC = PR [From (3)]
Therefore, by SSS congruency criterion, ΔABC ≅ ΔPQR
∴∠B = ∠Q (By CPCT)
However, ∠Q = 90º (By construction)
∴∠B = 90º
Hence, in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

26. Draw a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle.

Solution:

Let us assume that right ΔABC has base BC = 6 cm, side AB = 8 cm, and ∠B = 90°. Let ΔA'BC' have sides that are $\frac{3}{4}$ times those of ΔABC.
Now, ΔABC and ΔA'BC' can be drawn as follows:
(1) Draw a line segment BC = 6 cm. Draw a ray BX making 90° with BC.

(2) Draw an arc of 8 cm radius taking B as its centre to intersect BX at A. Join AC. ΔABC is the required triangle.
(3) Draw a ray BY making any acute angle with BC on the other side of line segment BC.
(4) Locate 4 points B_1, B_2, B_3, and B_4 on ray BY such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
(5) Join B_4C. Draw a line through B_3 parallel to B_4C intersecting BC at C'.
(6) Through C', draw a line parallel to AC intersecting AB at A'. ΔA'BC' is the required triangle.



27. Prove the following:
(1 + cot A − cosec A) (1 + tan A + sec A) = 2
OR
Prove the following:
sin A (1 + tan A) + cos A (1 + cot A) = sec A + cosec A

Solution:

$$\text{L.H.S}=(1+\cot A-\cos ecA)(1+\tan A+\sec A)$$

$$=\left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)\quad\left[\cot A=\frac{\cos A}{\sin A},\tan A=\frac{\sin A}{\cos A}\right]$$

$$=\left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$

$$=\frac{(\sin A+\cos A-1)(\sin A+\cos A+1)}{\sin A\cos A}$$

$$=\frac{(\sin A+\cos A)^2-1^2}{\sin A\cos A}\quad\left[(x+y)(x-y)=x^2-y^2\right]$$

$$=\frac{\sin^2 A+\cos^2 A+2\sin A\cos A-1}{\sin A\cos A}$$

$$=\frac{1+2\sin A\cos A-1}{\sin A\cos A}\quad\left[\sin^2 A+\cos^2 A=1\right]$$

$$\therefore(1+\cot A-\cos esA)(1+\tan A+\sec A)=2$$

OR

[[S]]

$$
\begin{aligned}
\text{L.H.S} &= \sin A(1+\tan A)+\cos A(1+\cot A)\\
&= \sin A\left(1+\frac{\sin A}{\cos A}\right)+\cos A\left(1+\frac{\cos A}{\sin A}\right)\quad\left[\tan A=\frac{\sin}{\cos A},\cot A=\frac{\cos A}{\sin A}\right]\\
&= \sin A\left(\frac{\cos A+\sin A}{\cos A}\right)+\cos A\left(\frac{\sin A+\cos A}{\sin A}\right)\\
&= (\cos A+\sin A)\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)\\
&= (\cos A+\sin A)\left(\frac{\sin^2 A+\cos^2 A}{\cos A\sin A}\right)\\
&= (\cos A+\sin A)\left(\frac{1}{\cos A\sin A}\right)\quad\left[\sin^2 A+\cos^2 A=1\right]\\
&= \frac{\cos A+\sin A}{\cos A\sin A}\\
&= \frac{1}{\sin A}+\frac{1}{\cos A}\\
&= \operatorname{cosec} A+\sec A\\
&= \sec A+\operatorname{cosec} A=\text{R.H.S.}
\end{aligned}
$$

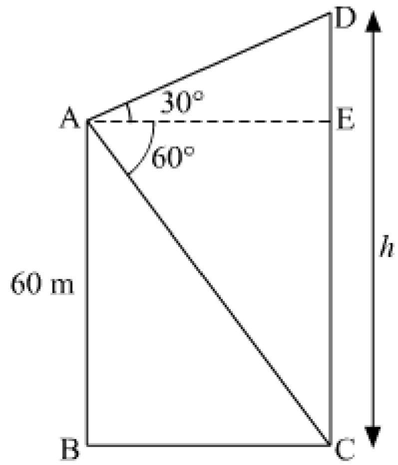
∴sin A(1 + tan A) + cos A(1 + cot A) = sec A + cosec A

28. The angles of elevation and depression of the top and bottom of a light-house from the top of a 60 m high building are 30° and 60° respectively. Find

(i) the difference between the heights of the light-house and the building.

(ii) the distance between the light-house and the building.

ANSWER



Let AB be the building and CD be the light house.
Suppose the height of the light house be *h* m.
Given: AB = 60 m, ∠EAD = 30° and ∠CAE = 60°.
CE = AB = 60 m
∴ DE = CD – CE = (*h* – 60) m
In ΔEAD,

$$\tan 30^\circ = \frac{DE}{AE}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{h-60}{AE}$$

$$\Rightarrow AE = \sqrt{3}(h-60) \qquad \ldots(1)$$

In ΔACE,

$$\tan 60^\circ = \frac{CE}{AE}$$

$$\therefore \sqrt{3} = \frac{60}{AE}$$

$$\Rightarrow AE = \frac{60}{\sqrt{3}} \qquad \ldots(2)$$

From (1) and (2), we get

$$\sqrt{3}(h-60) = \frac{60}{\sqrt{3}}$$

$$\therefore 3(h-60) = 60$$

$$\Rightarrow 3h - 180 = 60$$

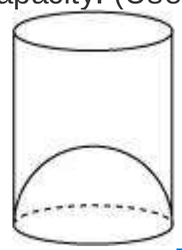
$$\Rightarrow 3h = 240$$

$$\Rightarrow h = 80$$

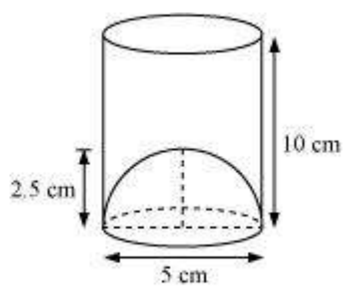
∴ **Difference between the height of light house and building = CD – AB = 80 m – 60 m = 20 m**

Distance between the light house and building = BC = AE $= \frac{60}{\sqrt{3}} = 20\sqrt{3}\text{m}$

29. A juice seller serves his customers using a glass as shown in Figure 6. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14)



Solution:



Apparent capacity of the glass will be the same as the capacity of the cylindrical portion having its base diameter as 5 cm and height as 10 cm.

Base radius = $\frac{5}{2} = 2.5 \text{ cm}$

Apparent capacity = $\pi r^2 h$

$= 3.14 \times (2.5)^2 \times 10$

$= 196.25 \text{ cm}^3$

Actual capacity of the glass will be the difference between the cylindrical portion and the hemispherical portion.

From the figure, it is clear that the base radius of the hemispherical portion is also $\frac{5}{2}$ cm or 2.5 cm.

Actual capacity of the glass = $\pi r^2 h - \frac{2}{3}\pi r^3$

$= 196.25 \text{ cm}^3 - \frac{2}{3} \times 3.14 \times (2.5)^3 \text{ cm}^3$

$= 196.25 \text{ cm}^3 - 32.71 \text{ cm}^3$

$= 163.54 \text{ cm}^3$

Thus, the apparent capacity of the glass is 196.25 cm^3 and the actual capacity of the glass is 163.54 cm^3.

30. A Group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest.
Which of the above values you prefer more?

Or

During the medical check-up of 35 students of a class their weights were recorded as follows:

Weight (in kg)	Number of students
38 − 40	3
40 − 42	2
42 − 44	4
44 − 46	5
46 − 48	14
48 − 50	4
50 − 52	3

Draw a less than type and a more than type ogive from the given data. Hence obtain the median weight from the graph.

Solution:

Since the group consists of 12 persons, sample space consists of 12 persons.

∴Total number of possible outcomes = 12

Let A denote event of selecting persons which are extremely patient

∴ Number of outcomes favourable to A is 3.

Let B denote event of selecting persons which are extremely kind or honest.

Number of persons which are extremely honest is 6.

Number of persons which are extremely kind is 12 − (6 + 3) = 3

∴ Number of outcomes favourable to B = 6 + 3 = 9.

(i) Probability of selecting a person who is extremely patient is given by P(A).

P(A) = $\frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{3}{12} = \frac{1}{4}$.

(ii) Probability of selecting a person who is extremely kind or honest is given by P(B)

P (B) = $\frac{\text{Number of outcomes favourable to B}}{\text{Total number of possible outcomes}} = \frac{9}{12} = \frac{3}{4}$

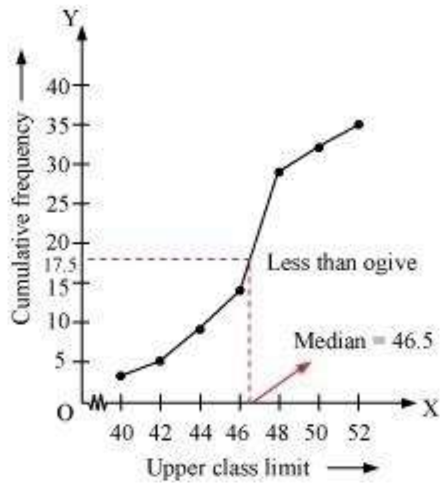
Or

For the given data, the cumulative frequency distribution of the less than type can be computed as follows.

Weight (in kg)	Number of students (Cumulative frequency)
Less than 40	3
Less than 42	3 + 2 = 5
Less than 44	5 + 4 = 9
Less than 46	9 + 5 = 14
Less than 48	14 + 14 = 28
Less than 50	28 + 4 = 32
Less than 52	32 + 3 = 35

To draw a less than ogive, we mark the upper class limits of the class intervals on the x-axis and their corresponding cumulative frequencies on the y-axis by taking a convenient scale.

Now, plot the points corresponding to the ordered pairs [(upper class limit, cumulative frequency) − i.e., (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32), (52, 35)] on the graph paper as follows:

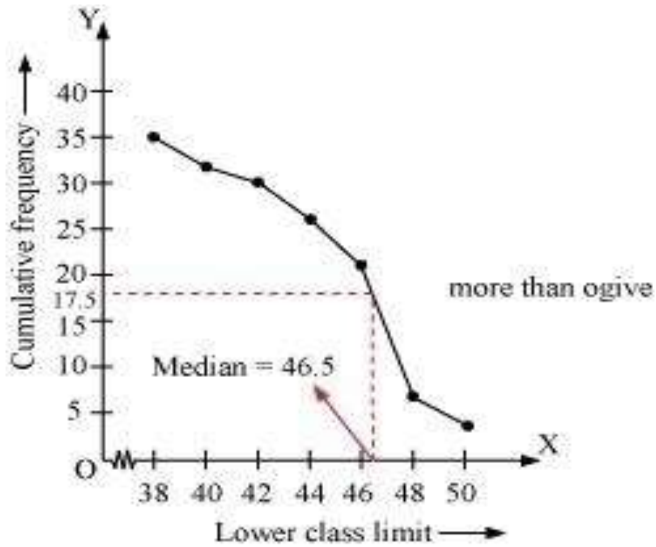


Similarly, we can compute the cumulative frequency distribution of the more than type as follows:

Weight (in kg)	**Number of students (Cumulative frequency)**
More than 38	35
More than 40	35 − 3 = 32
More than 42	32 − 2 = 30
More than 44	30 − 4 = 26
More than 46	26 − 5 = 21
More than 48	21 − 14 = 7
More than 50	7 − 4 = 3

Now, to draw a more than ogive, we mark the lower class limits of the class intervals on the *x*-axis and their corresponding cumulative frequencies on the *y*-axis by taking a convenient scale.

Now, plot the points corresponding to the ordered pairs [(lower class limit, cumulative frequency) − i.e., (38, 35), (40, 32), (42, 30), (44, 26), (46, 21), (48, 7), (50, 3)] on the graph paper as follows:



Now, to obtain the median weight from the graph, we draw both ogives on the same graph paper. They intersect at (46.5, 17.5). 46.5 kg is the median weight of the given data.