CLASS XII
GUESS PAPER
MATHEMATICS

TIME- 3 HOURS M.M-100

General Instructions:

The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.

Section-A

1. If f(x) = $[x]$ and g(x) =$|x|$, $x \in R$, find (fog) $(-\frac{5}{2})$.
2. If A is a square matrix of order 3x3 and $|A| = 5$, find $|Adj\ A|$
3. Evaluate $\int \frac{\tan x}{\sec^2 x} dx$
4. What is the principal value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$.
5. If A=$\begin{bmatrix} 7 & -4 \\ 2 & 3 \end{bmatrix}$, find det$((A)^{-1})$.
6. Find a, for which f(x) = a (x + sinx) +a is increasing
7. Find order and degree of differential equation $\frac{dy}{dx} + \frac{3x}{\frac{dy}{dx}} + 4y = 2xy$
8. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.
9. Find the unit vector perpendicular to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 2\hat{k}$.
10. Find direction ratios of line $\frac{1-x}{2} = \frac{2y+3}{5}, z = 3.$

[SECTION – B]

11. Find the equation of the tangent to the curve y = cos (x + y), $-2\pi \le x \le 2\pi$ that are parallel to the line x + 2y = 0
12. If y = x log $\left(\frac{x}{a+bx}\right)$, Prove that $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{a}{a+bx}\right)^2$.
13. Let A= $\{-1,0,1,2\}, B = \{-4,-2,0,2\}$and $f, g : A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A.$ Are f and g equal? Justify your answer.
14. Evaluate: $\int \frac{\tan^{-1} x}{(1+x)^2} dx$

OR

Evaluate $\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$

15 If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, Prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

OR

If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$ Prove that $\frac{dy}{dx}=\frac{1}{x^3y}$

16. If a,b and c are all positive and different, prove that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is always negative.

17. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

18. Solve the differential equation: $x\cos\left(\frac{y}{x}\right).(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$

19. Solve the following differential equation: $Sin^{-1}\left(\frac{dy}{dx}\right)=x+y$

20. If the sum of two unit vectors is a unit vector; show that magnitude of their difference is $\sqrt{3}$.

21. Solve for x : $sin^{-1}(1-x)-2sin^{-1}x=\frac{\pi}{2}$

OR

Prove that $\tan(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b})+\tan(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b})=\frac{2b}{a}$

22. Show that the lines $x=ay+b, z=cy+d$ *and* $x=a'y+b', z=c'y+d'$ are perpendicular to each other, if $aa'+cc'+1=0$.

[SECTION – C]

23. Find the image of point (3, 5, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3.}$.

24. Find the area of the region $\{(x,y): x^2+y^2\le 2ax, y^2\ge ax, x\ge 0, y\ge 0\}$

25. Evaluate: $\int_0^1 \cot^{-1}(1-x+x^2)dx$

OR

Evaluate the following integral as limit of sum $\int_1^3 e^{2x-1}.dx$

26. If A = $\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1}, solve the equations: $2x-3y+5z=11, 3x+2y-4z=-5, x+y-2z=-3$

27. A factory owner purchases two types of machines, A and B, for his factory. The requirements and limitations for the machines are as follows:

	Area occupied by the machine	Labour force for each machine	Daily output in units
Machine A	1000 sq. m	12 men	60
Machine B	1200 sq. m	8 men	40

He has an area of 9000 sq. m available and 72 skilled men who can operate machines. How many machines of each type should he buy to maximize daily output**?**

28. A card is lost from a pack of 52 cards. From the remaining cards of the pack, two cards are drawn and are found to be diamonds. What is the probability that the lost card is a spade?

29. A window is in the form of a rectangle above which there is a semicircle. If the perimeter of the window is p cm. Show that the window will allow the maximum possible light only when radius of the semicircle is $\frac{p}{\pi+4}cm$.

OR

Find the equation of the normal at any point θ to the curve $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$. Also show that the normal is at a constant distance from the origin.

GUESS PAPER-2, MATHEMATICS-XII

TIME- 3 HOURS M.M-100

Section-A

1. Let * be the binary operation on N defined by a * b = H.C.F. of a and b. Find 4*6.
2. If A is square matrix of order 3 such that $|adjA|=81$, find the value of $|A|$.
3. Evaluate $\int_{-1}^{1} x^{17}\cos^4 x dx$
4. Find the principal value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)+\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$
5. If A is a square matrix satisfying A^2 =I, then find inverse of A.

6. At what point on the curve $y^2 = 2x - 1$ does the ordinate decrease at same rate as the abscissa increases?

7. Find order and degree of differential equation $\sqrt{x\frac{d^2y}{dx^2} + 2y\frac{dy}{dx}} = y\frac{d^3y}{dx^3}$

8. For what value of μ are the vectors $\vec{a} = 2\hat{i} + \mu\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

9. Find the angle between two vectors $\vec{a}$ and $\vec{b}$. if $|\vec{a} \times \vec{b}| = \vec{a}.\vec{b}$

10. Find the vector normal to the plane x + 2y + 3z – 6 = 0

[SECTION – B]

11. A balloon which always remains spherical is being inflated by pumping in 900 cc of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

OR

Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points P (2, 0) and Q (3, 0) are at right angles.

12. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, Prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

13. Show that the function f: R→R defined by f(x) = 3x+4 is invertible. Also find $f^{-1}(x)$.

14. Evaluate: $\int \sin 4x \cos 7x dx$

15. If f(x), defined by following, is continuous at x=$\frac{\pi}{2}$, find the values of a and b, f(x)

$$\begin{cases} \frac{1 - \sin^3 x}{3\cos^2 x} & if\ x < \frac{\pi}{2} \\ a & if\ x = \frac{\pi}{2} \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2} & if\ x > \frac{\pi}{2} \end{cases}$$

OR

If $(x-a)^2 + (y-b)^2 = c^2$, *for* some c > 0, Prove that $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a & b.

16. Using properties of determinants, prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

17. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he fire so that the probability of hitting the target at least once is more than 0.99?

18. Solve the differential equation: $\left(x^2 - y^2\right)dx + 2xydy = 0;\ y(1) = 1$

OR

Solve the following differential equation: $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$

19. Solve the differential equation: $\log\left(\frac{dy}{dx}\right) = ax + by$.

20. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then show that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$

21. Solve for x: $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

22. Show that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar. Also find the equation of the plane containing these lines.

[SECTION – C]

23. Prove that the image of the point (3, –2, 1) in the plane 3*x* □□*y* □□4z □□2 lies on the plane *x* □□*y* □□z □□4 □□0.

OR

Find the equation of the plane passing through the intersection of the planes, 2x+3y+z=1, x+y+2z=3 and perpendicular the plane 3*x* □+□*y* □+□2z □=□4. Also find the inclination of this plane with the *xy* plane.

24. Find the area of the region $\{(x, y) : |x-1| \le y \le \sqrt{5 - x^2}\}$

25 valuate: $\int \frac{1}{\sin x(5 - 4\cos x)}dx$

26. $If\ A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}. Prove\ that\ A^2 - 4A - 5I = 0. Hence\ find\ A^{-1}.$

OR

Using matrices solve the following system of equations:

$x + \frac{2}{y} + 3xz = -1,\qquad 2x - \frac{4}{y} - 3xz = 3,\qquad 3x + \frac{6}{y} - 2xz = 4$

27. A firm manufactures two products, X and Y,each requiring the use of three machines M_1, M_2 and M_3. the time required for each product in hours and total time available in hours on each machine are as follows:

Machine	Product X	Product Y	Available time(in hours)
M_1	2	1	70
M_2	1	1	40
M_3	1	3	90

If the profit is Rs 40 per unit for product X and Rs 60 per unit for product Y, how many units of each product should be manufactured to maximize profit?

28. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output, 5, 4 and 2 percent are defective bolts respectively. A bolt is drawn at random from the product.
    (i) What is the probability that the bolt drawn is defective?
    (ii) If the bolt is found to be defective, find the probability that it is a product of machine B.

29. Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex coinciding with one extremity of the major axis.