

THE VELAMMAL INTERNATIONAL SCHOOL
2010-11 MODEL PAPER - I
XII STD MATHEMATICS

SECTION – A (10 X 1 = 10 MARKS)

- Write the number of all one to one functions from the set $A = \{ a, b, c \}$ to itself.
- Write the range of the principal value branch of $\operatorname{cosec}^{-1}x$ defined on the domain $R - (-1,1)$.
- State the number of all possible matrices of order 3×2 with each entry 2 or 1.

4. Evaluate using properties of determinants
$$\begin{vmatrix} \sqrt{13} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{26} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix}.$$

5. Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$

6. If $f(x) = \tan 3x^\circ$ find $\frac{dy}{dx}$

7. Find the order and degree of the differential equation $\sin^{-1}\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right) = \frac{\pi}{3}$

8. If \vec{p}, \vec{q} and $\vec{p} + \vec{q}$ are vectors of magnitude λ , then find $|\vec{p} - \vec{q}|$.

9. Find the magnitude and direction cosines of $2\vec{a} + 3\vec{b}$ where $\vec{a} = \vec{i} - 3\vec{j} + 4\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} + 3\vec{k}$

10. Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{-2}$ and the plane $3x+4y+z+5=0$.

SECTION – B (12 X 4 = 48 MARKS)

- Consider $f:N \rightarrow N$, $g:N \rightarrow N$ and $h:N \rightarrow R$ where $f(x) = 2x$, $g(y) = 3y+4$, $h(z) = \sin z$, $\forall x,y,z \in N$. Show that $h \circ (g \circ f) = (h \circ g) \circ f$.
- Find the value of $\tan^{-1} \frac{1}{2} \left(\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right)$, $|x| < 1$, $y > 0$, $xy < 1$

13. Show that
$$\begin{vmatrix} a+b+c & -c & -b \\ -c & b+c+a & -a \\ -b & -a & c+a+b \end{vmatrix} = 2(a+b)(b+c)(c+a)$$
14. Discuss the differentiability of $f(x) = |x-1| + |x-2|$.
15. If $y\sqrt{x^2+1} = \log\left[\sqrt{x^2+1}-x\right]$, Show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$
16. Find 'c' of Rolle's theorem for $f(x) = x^3 - 9x^2 + 26x - 24$ in $[2, 4]$
17. Evaluate $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}} dx$ (OR) $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$
18. Evaluate $\int \frac{\sqrt{1-\sqrt{x}}}{1+\sqrt{x}} dx$
19. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$ (OR) $\int \frac{\sin(x)}{\sin(x-\alpha)} dx$
20. If \vec{a}, \vec{b} and \vec{c} are three mutually perpendicular vectors of equal magnitude, prove that the angle which $(\vec{a} + \vec{b} + \vec{c})$ makes with any of the vectors \vec{a}, \vec{b} or \vec{c} is $\cos^{-1} \frac{1}{\sqrt{3}}$
21. Find the vector and Cartesian equation of the plane passing through $(-1, 3, 2)$ point and perpendicular to the planes $x+2y+3z=5$ and $3x+3y+z=0$
22. A bag contains 4 balls, two balls are drawn at random and found to be white. What is the probability that all are white.

SECTION – C (7X 6 = 42 MARKS)

23. Solve the system of equations $2x-3y+5z = 11$; $3x+2y-4z = -5$; $x+y-2z = -3$.
24. An open tank with square base and vertical sides is to be constructed from metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when the depth of the tank is half the width.
25. Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-2)^2+y^2=4$.
26. Solve $y + \frac{d(xy)}{dx} = x(\sin x + \log x)$

27. If a variable plane at a constant distance p from the origin meets the co-ordinate axes in points A, B and C respectively. Through these points, planes are drawn parallel to the co-ordinate planes. Show that the locus of the point of intersection is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.
28. Find the mean, variance and standard deviation of the number of heads in tossing a biased coin thrice such that heads appear twice as tail appears.
29. Anil wants to invest at most Rs.12000 in Bonds A and B. According to the rules, he has to invest at least Rs.2000 in Bond A and at least Rs.4000 in bond B. If the rate of interest on Bond A is 8% per annum and on bond B is 10% per annum, how should he invest his money for maximum interest? Solve the problem graphically.

PREPARED BY

**D. VASU RAJ M.Sc., B.Ed.,
31/15, EAST MADA STREET,
VADAPALANI,
CHENNAI - 600026.
TAMIL NADU**

EMAIL : vraj626@yahoo.co.in