ARMY SCHOOL JAMMUCANTT

PRE BOARD EXAMINATION 2010-11

CLASS: XII **M.M : 100**

SUBJECT: MATHEMATICS **TIME: 3 HRS**

GENERAL INSTRUCTION:

(a) *All questions are compulsory.*
(b) *This question paper consists of 29 questions divided into three section A, B, and C. Section A comprises of 10 question of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.*
(c) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
(d) *There is no overall choice. However, internal choice has been provided in 03 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
(e) *Use of calculators is not permitted. You may ask for logarithmic tables, if required.*

SET -I

SECTION A

Q1 If A is a square matrix of order 3 such that |adj A| = 64, find |A|.

Q2. If A, B, C are three non zero square matrices of same order, find the condition on A such that AB = AC $\Longrightarrow$ B = C

Q3. Give an example of two non zero 2×2 matrices A, B such that AB = 0.

Q4. If f(1) = 4; $f'(1) = 2$, find the value of the derivative of log $f(e^x)$ *w.r.t.* x at the point $x = 0$.

Q5. Find a, for which f(x) = a(x + sin x) + a is increasing.

Q6. Evaluate : $\int_0^{1.5}[x]dx$, (where [x] is greatest integer function)

Q7. Write a value of $\int e^{3logx}(x^4)$dx..

Q8. If $\vec{a} = \hat{\imath} + \hat{\jmath}$; $\vec{b} = \hat{\jmath} + \hat{k}$; $\vec{c} = (\hat{k} + \hat{\imath})$, find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$.

Q9. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1 : 4, where $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\hat{b} = -\hat{\imath} + \hat{\jmath} + \hat{k}$.

Q10. The probability that an event happens in one trial of an experiment is 0.4. Three independent trails of the experiment are performed. Find the probability that the event happens at least once.

SECTION – B

Q11: Show that the function f : R→ R defined by f (x) = $\frac{2x-1}{3}$, x ϵ R is is one-one and onto function. Also find the inverse of the function f.

OR

Examine which of the following is a binary operation:

(i) a * b = $\frac{a+b}{2}$, a, b ϵ N (ii) a * b = $\frac{a+b}{2}$, b, ∉Q

for binary operation check the commutative and associate property.

Q12: Prove that $\tan^{-1}\left(\frac{63}{16}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$.

Q13: Using properties of determinants, prove that:

$$\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (ab + bc + ca)^3.$$

Q14: Find all the points of discontinuity of the function *f* defined by

$$f(x) = \begin{cases} x + 2, & x \le 1 \\ x - 2, & 1 < x < 2 \\ 0, & x \ge 2 \end{cases}$$

Q15: If $x^p y^q = (x + y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

OR

Find $\frac{dy}{dx}$, if y = $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, 0 < |x| < 1.

Q16. Evaluate $\int \frac{(x^2+1)\,(x^2+4)}{x^2+3)\,(x^2-5)}$ dx. OR $\int \frac{1}{x^2\,(x^4+1)^{3/4}}\,dx$

Q17. A water tan has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$ water is poured into it at a constant rate of 5 cubic meter per minute.

Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m.

Q18. Evaluate the following integral as limit of sum $\int_1^2 (3x^2 - 1)dx$

Q19. Evaluate $\int_0^{\frac{\pi}{2}} \log sinx\, dx$

Q20. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through (3, 0, 4). Also find the distance between these two lines.

Q21. Show that the points A, B, C with position vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ respectively are the vertices of a right angled triangle. Hence find the its area ..

Q22. A football match may be either won, drawn or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

SECTION C

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Q23. A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 2$

OR

Obtain the inverse of the matrix A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operations

Q24. Find all the local maximum values and local minimum values of the function f(x) = sin 2x –x. $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Or

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$, Also, find the maximum volume.

Q25. Find the area of the region enclosed between two circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

Q26. Solve the following differential equation $(1-x^2)\frac{dy}{dx} - xy = x^2$, given by $y = 2$ when $x = 0$.

Q27. Find the equation of the plane passing through the intersection of the planes, $2x+3y-z+1=0$; $x + y - 2z = 0$ and perpendicular to the plane $3x - y - 2z - 4 = 0$. Also find the distance of this plane from origin.

Q28. In a test, an examine either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$, and the probability that he copies is $\frac{1}{6}$. The probability that his answer is correct $\frac{1}{8}$, given that the copies is . Find the probability that the he knew the answer, given that he correctly answered it.

Q29. A hospital dietician wishes to find the cheapest combination of two foods. A and B, that contains at least 50 milligram of thiamine and at least 600 calories. Each unit of A contains 12 milligram of thiamine and 100 calories, while each unit of B contains 10 milligram of thiamine and 150 calories. If each food costs Rs. 10 per unit, how many units of each should be combined to minimise the cost?

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SET-II

SECTION-A

Q1. Find the cartesian equation of a line which passes through the point (1, 2, 3) and parallel to the line $\frac{-x-2}{1} - \frac{y+3}{7} = \frac{2z-6}{3}$.

Q2. Find the domain of the function : $\sqrt{(x-1)(3-x)}$

Q3. Simplify : $\sin^{-1}\left[\frac{sinx+cosx}{\sqrt{2}}\right]$; $-\frac{\pi}{4} < x < \frac{\pi}{4}$

Q4. If A = $\begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$, show that $A^T A = 1$

Q5. If the vectors $2\hat{\imath} - 5\hat{\jmath} + b\hat{k}$ and $2\hat{\imath} - aj + 4\hat{k}$ are parallel, find the value of a and b.

Q6. Find the general solution of $\frac{dy}{dx} = \frac{x-y+1}{x-y}$

Q7. Solve : [x 1] $\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$

Q8. Using determinants, show that (1, 4) (2, 3) and (– 2, 7) are collinear.

Q9. Show that y – λx4 is a solution of the differential equation $x\frac{dy}{dx} - 4y = 0$.

Q10. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 6$. Find $|\vec{a} + \vec{b}|$.

SECTION-B

Q11. Evaluate: $\int \frac{18}{(x+2)(x^2+4)}$ dx.

Q12. Scalar products of a vector with vectors $3\hat{\imath} - 5\hat{k},\ 2\hat{\imath} + 7\hat{\jmath}$ and $\hat{\imath} + \hat{\jmath} + \hat{k}$ are respectively –1, 6 and 5. Find the vector.

Q13. If sin y = x sin (a + y), prove that = $\frac{dy}{dx} = \frac{sin^2\,(a+y)}{\sin a}$.

Q14. Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}\frac{3}{5}$

Q15. Using properties of determinants prove that $\begin{bmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{bmatrix} = 0$

Or

Prove that $\begin{bmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{bmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

Q16. If $y = \tan^{-1}\left(\frac{5ax}{a^2-6x^2}\right), prove\ that$

$$\frac{dy}{dx} = \frac{3a}{a^2+9x^2} + \frac{2a}{a^2+4x^2}$$

Or

If $y = \left\{x + \sqrt{x^2 + a^2}\right\}^n$ Prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$.

Q17. Evaluate : $\int_0^{\frac{\pi}{4}} \log(1 + tanx)\, dx$

Or

Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{sinx+cosx}{\sqrt{sin2x}}\, dx$

Q18. Find the equation of the plane passing through (2, 2,1) and (9, 3, 6) and is perpendicular to the plane $2x + 6y + 6z = 1$

Q19. Form the differential equation of family of circles touching x axis at origin .

Q20. Solve : $\sqrt{1+ x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$

Q21. A and B take turn in throwing two dice. The first to throw a sum 9 being awarded. Show that if A has the first throw, their chances of winning are in the ratio 9:8.

Q22. Let A = N× N and * be the binary operation on A defined by (a, b)*(c, d) = (a +c, b + d) Show that * is commutative and associative. Find the identity element for * on A, if any.

SECTION-C

Q23. Find the equation of the perpendicular drawn from the point (2, 4,–1) to the line $\frac{x+5}{1} + \frac{y+3}{4} = \frac{z-6}{-9}$. Also find its image.

Q24 If A= $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ then find inverse of A using elementary transformations .

OR

Gaurav purchased 3 pens, 2 bags, 1 instrument box for Rs. 41. Dheeraj purchased 2 pens, 1 bag and 2 instrument boxes for Rs. 29; while Ankur purchased 2 pens, 2 bags and 2 instrument boxes for Rs. 44. Translate the problem into a system of equations. Solve using matrix method and hence, find the cost of 1 pen, 1 bag and 1 instrument box.

Q25: A point on the hypotenuse of a triangle is at distance a and b from the sides. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

OR

Find the intervals on which the function $f(x) = (x+1)^3 (x-3)^3$ is strictly increasing and strictly decreasing.

Q26: Using integration, find the area of the triangle whose vertices are A(0, 5), B (–1, 1) and C(3,2)

Q27: Evaluate : $\int (\sqrt{\cot x} + \sqrt{\tan x})\, dx$

Q28: One card is lost from the pack of 52 cards . Two cards are drawn from the remaining cards and are found to be both diamonds ,find the probability of lost card being diamond. .

Q29: A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 5% phosphoric acid, and type B which contains 6% nitrogen and 10% phosphoric acid. After testing the soil conditions of the field, it was found that at least 14 kg of nitrogen and 14 kg of phosphoric acid is required for producing a good crop. The fertilizer of type A costs Rs 5 per kg and the type B costs Rs 3 per kg. How many kg of each type of the fertilizer should be used to meet the requirement at the minimum possible cost? Using L.P.P. solve the above problem graphically.