Paper with COMPLETE SOLUTIONS & MARKING SCHEMES according to the Syllabus & Guide Lines of CBSE BOARD

CLASS-XII (2016-2017)

QUESTION WISE BREAK UP

Type of Question	Mark per Question	Total No. of Questions	Total Marks
VSA	1	04	04
SA	2	08	16
LA-I	4	11	44
LA-II	6	06	36
	Total	26	100

CHAPTERWISE MARKS in Class-XII (CBSE)

Sr. No	TOPICS	MARKS					
		VSA(1M)	SA(2M)	S A (4M)	L A (6M)	Total Marks	
1 a	Relation & Function	1			1 OR 1	8	10
1 b	Binary operation	1					
1 c	Inverse Trig. Func		1			2	
2.a	Matrices	1	1	1		7	13
b	Determinant				1 OR 1	6	
3.a.	Continuity, Differentiability, DERIVATIVE		1	1 OR 1 +1		10	10
b.	Applications Of Derivative		1	1 OR 1 +1		10	10
c.	Integrals		1	1	1 OR 1	12	12
d	Applications Of Integrals				1	6	6
e	Differential Equations		1	1 OR 1		6	6
4.a	Vectors	1	1			3	3
b	Three Dimensional Geometry			1+1	1	14	14
5.	Linear Programming				1	6	
6.	Probability		1	1+ 1 (VBQ)		10	
	TOTAL	04	08	11	6	100	

[Mp 01 Cbse Xii 01 Q&H 162112] MODEL TEST [FM–100/Time–3 hrs.]
[Pre CBSE-XII'17]

GENERAL INSTRUCTIONS:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

SECTIONS – A (Questions 01 to 04 carry 1 marks each)

1. Let $*$ be the binary operation on **N** defined by $a * b$ = H.C.F. of a and b. Is $*$ commutative? Justify.
2. If A is a square matrix of order 3 and $|3A| = k|A|$, then find the value of k.
3. Are the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ collinear or not.
4. The function $f: N \to N$ is defined by $f(n): \begin{cases} \frac{n+1}{2}, & n \text{ is odd} \\ \frac{n}{2}, & n \text{ is even} \end{cases} \quad \forall n \in N$. State whether the function is one-one or not.

SECTIONS – B (Questions 05 to 12 carry 2 marks each.)

5. Find the principal value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$.
6. If $A = \begin{bmatrix} 3 & -5 \\ 0 & 1 \end{bmatrix}$ and $A - B - 3.I_2 = O$ (O is a null matrix of order 2), find the matrix B.
7. If $y = \tan^{-1}\left(\frac{7x}{1-12x^2}\right)$, $-\frac{1}{2\sqrt{3}} < x < \frac{1}{2\sqrt{3}}$, then prove that, $\frac{dy}{dx} = \frac{4}{1+16x^2} + \frac{3}{1+9x^2}$.
8. Using differentials, find the approximate value of $\sqrt[5]{32.15}$ up to 3 places of decimal.
9. Evaluate : $\int \left(\frac{e^x - 1}{e^x + 1}\right) dx$
10. Obtain differential equation of the family of circles passing through the points (0, a) and (0, – a).
11. A and B are two points whose position vectors are $(2\vec{a}-\vec{b})$ and $(\vec{a}+2\vec{b})$ respectively. If the point C divides the joining of points A and B externally in the ratio of 2 : 1, then find position vector of C.
12. Given $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cap \bar{B}) = \frac{1}{6}$. Test whether A and B are independent events or not.

SECTIONS – C (Questions 13 to 23 carry 4 marks each.)

13. Express the matrix $A = \begin{bmatrix} 3 & 4 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

14. If f(x) is differentiable at x = a, then find $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x-a}$.

OR, Find the value of k so that the function f defined by $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

15. If $2x = y^{\frac{1}{n}} + y^{\frac{-1}{n}}$ then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = n^2 y^2$.

16. Find the equation of the normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (a secθ , b tanθ).

OR, Separate the interval $\left[0, \frac{\pi}{2}\right]$ into two sub-intervals in which the function $f(x) = \sin^3 x + \cos^3 x$ is strictly increasing or strictly decreasing.

17. The sum of the surface area of a cube and a sphere is constant. Find the ratio of the edge of the cube to the diameter of the sphere, when the sum of their volumes is minimum.

18. Evaluate : $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$.

19 Find the general solution of the differential equation $(2x - 10y^3)\frac{dy}{dx} + y = 0$

OR Solve : $y.\cos\frac{y}{x}(x\,dy - y\,dx) + x.\sin\frac{y}{x}(x\,dy + y\,dx) = 0$, if $y(1) = \frac{\pi}{2}$.

20. Find the area of the triangle whose vertices are A(3, – 1, 2), B(1, – 1, –3) and C(4, –3, 1).

21. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 3\hat{k}$ in the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

22. An insurance company insured 400 cyclists, 8000 scooter drivers and 12000 car drivers. The probability of an accident involving a cyclist, scooter driver and a car driver are 0.02, 0.06 and 0.30 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver ? Which mode of transport would you suggest to a student and why ?

23. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

SECTIONS – D (Questions 24 to 29 carry 6 marks each)

24. Show that function f: $\mathbf{R} \to \{x \in \mathbf{R}: -1 < x < 1\}$ defined by $f(x) = \dfrac{x}{1+|x|}$, $x \in R$ is one-one and onto function.

OR, Define a binary operation *on the set {0, 1, 2, 3, 4, 5} as $a * b = \begin{cases} a+b, & if\ a+b < 6 \\ a+b-6, & if\ a+b \geq 6 \end{cases}$

Show that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with $6 - a$ being the inverse of a.

25. Using properties of determinant, prove that, $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$.

OR, If a, b, c, are in A.P., then using properties of determinant, find the value of $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$.

26. Find the area of the region $\{(x, y): x^2 \leq y \leq |x|\}$

27. Prove that, $\int_0^{\frac{\pi}{2}} \dfrac{\sin x.\cos x}{a^2.\cos^2 x + b^2.\sin^2 x} dx = \dfrac{\log(a/b)}{a^2 - b^2}$ Given $a > 0, b > 0, a \neq b$

OR, Evaluate $\int_1^2 (2x+5)\, dx$ as the limit of a sum.

28. If straight lines having direction cosines given by al + bm + cn = 0 and fmn + gnl + hlm = 0 are perpendicular, then show that $\dfrac{f}{a} + \dfrac{g}{b} + \dfrac{h}{c} = 0$.

29. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of `12 and `16 per doll respectively on dolls A and B. Formulate the problem as LPP and solve it graphically to find how many of each should be produced weekly in order to maximize the profit ?

"*The only way to learn* ***MATHEMATICS*** *is to do* ***MATHEMATICS***." – Paul Halmos.

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HINTS with Step Markings

SECTIONS – A **(Questions 01 to 04 carry 1 marks each)**

1. HINTS: $a * b$ = H.C.F. of a and b = H.C.F. of b and $a = b * a$. Hence $*$ is commutative. [1]

2. HINTS: k= $3^3 = 27$. [1]

3. HINTS: $\because -2\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=\left(-4\hat{i}+6\hat{j}-8\hat{k}\right)$. Hence the vectors are collinear. [1]

4. HINTS: $f(1)=\frac{1+1}{2}=1,\ \ f(2)=\frac{2}{2}=1$ [By definition] $\therefore f(1) = f(2) = 1$. So f is not one-one. [1]

SECTIONS – B **(Questions 05 to 12 carry 2 marks each.)**

5. HINTS: $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)=\tan^{-1}\left(\tan\left(\pi+\frac{\pi}{8}\right)\right)$ [1]

$=\tan^{-1}\left(\tan\frac{\pi}{8}\right)=\frac{\pi}{8}\quad\left[\because -\frac{\pi}{2}<\tan^{-1}x<\frac{\pi}{2}\right]$ [1]

6. HINTS: $\because A=\begin{bmatrix}3 & -5\\ 0 & 1\end{bmatrix}$ and $A-B-3I_2=O\quad \therefore -B=O+3I_2-A$

$\Rightarrow -B=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}+3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}3 & -5\\ 0 & 1\end{bmatrix}\quad \Rightarrow -B=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}-\begin{bmatrix}3 & -5\\ 0 & 1\end{bmatrix}$ [1]

$\Rightarrow -B=\begin{bmatrix}0+3-3 & 0+0-(-5)\\ 0 & 0+3-1\end{bmatrix}\quad \Rightarrow -B=\begin{bmatrix}0 & 5\\ 0 & 2\end{bmatrix}\quad \therefore B=\begin{bmatrix}0 & -5\\ 0 & -2\end{bmatrix}$ [1]

7. HINTS : $y=\tan^{-1}\left(\frac{4x+3x}{1-4x.3x}\right)=\tan^{-1}4x+\tan^{-1}3x$ [1]

$\Rightarrow \frac{dy}{dx}=\frac{4}{1+(4x)^2}+\frac{3}{1+(3x)^2}=\frac{4}{1+16x^2}+\frac{3}{1+9x^2}$ [1]

8. HINTS : Consider, $y=x^{\frac{1}{5}}\quad x=32$ and $\Delta x=0.15$.

$\Delta y=\left(x+\Delta x\right)^{\frac{1}{5}}-x^{\frac{1}{5}}=(32.15)^{\frac{1}{5}}-32^{\frac{1}{5}}=(32.15)^{\frac{1}{5}}-2$ [$\frac{1}{2}$]

$\Rightarrow\ (32.15)^{\frac{1}{5}}=2+\Delta y$

Now, dy is approximately equal to Δy and is given by,

$$\Rightarrow dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{5x^{\frac{4}{5}}}.(\Delta x) \qquad \left[as\ y = x^{\frac{1}{5}}\right]$$ [1]

$$= \frac{1}{5.2^4}.(0.15) = \frac{0.15}{80} = 0.00187$$

Hence, the approximate value of $\sqrt[5]{32.15}$ is 2+0.00187 = 2.00187. [$\frac{1}{2}$]

9. HINTS: $\int\left(\frac{e^x-1}{e^x+1}\right)dx = \int\left(1+\frac{-2}{e^x+1}\right)dx = \int dx + 2\int\frac{-e^{-x}}{1+e^{-x}}dx$ [$\frac{1}{2}$]

$= x + 2\log\left|1+e^{-x}\right| + C$ [C is constant of integration] $\left[\because d(1+e^{-x}) = -e^{-x}dx\right]$ [1]

$= x + 2\left[\log\left|1+e^x\right| - x\right] + C = -x + 2\log\left|e^x+1\right| + C$ [$\frac{1}{2}$]

10. HINTS: The centre of the family of circles passing through (0, a) and (0, – a) must lie on the y-axis. Let the coordinates of the centre be (k, o).

The equation of the circles can be written as $(x-k)^2 + y^2 = k^2 + a^2 \Rightarrow x^2 + y^2 - 2kx - a^2 = 0$ … (i) [$\frac{1}{2}$]

Differentiating both sides of (i) w.r.to x, $2x + 2y\frac{dy}{dx} - 2k = 0 \quad \Rightarrow x + y\frac{dy}{dx} = k \quad \cdots(ii)$ [$\frac{1}{2}$]

By eliminating k from equations (i) & (ii), we get t required differential equation as

$x^2 + y^2 - a^2 - 2x^2 - 2xy\frac{dy}{dx} = 0 \quad \Rightarrow 2xy\frac{dy}{dx} + x^2 - y^2 + a^2 = 0$ [1]

11. HINTS: The position vector of the point C is $\frac{2\times\left(\vec{a}+2\vec{b}\right) - 1\times\left(2\vec{a}-\vec{b}\right)}{2-1}$ [1]

$= \frac{2\vec{a} + 4\vec{b} - 2\vec{a} + \vec{b}}{2-1} = 5\vec{b}$ [1]

12. HINTS : $P(B) = \frac{1}{3} \quad \Rightarrow P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$ [$\frac{1}{2}$]

$\Rightarrow P(A).P(\bar{B}) = \frac{1}{\cancel{4}_2}.\frac{\cancel{2}}{3} = \frac{1}{6} = P(A \cap \bar{B})$ [1]

$\therefore$ A & B are independent. [$\frac{1}{2}$]

SECTIONS – C (Questions 13 to 23 carry 4 marks each.)

13. HINTS : $A=\begin{bmatrix}3&4&-1\\-2&-2&1\\-4&-5&2\end{bmatrix} \Rightarrow A'=\begin{bmatrix}3&-2&-4\\4&-2&-5\\-1&1&2\end{bmatrix}$ [$\frac{1}{2}$]

$$P=\frac{1}{2}(A+A')=\frac{1}{2}\left[\begin{bmatrix}3&4&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}+\begin{bmatrix}3&-2&-4\\4&-2&-5\\-1&1&2\end{bmatrix}\right]=\begin{bmatrix}3&1&-\frac{5}{2}\\1&-2&-2\\-\frac{5}{2}&-2&2\end{bmatrix}=P'$$

Hence P is a symmetric matrix. [1]

Again $Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}3&4&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}-\begin{bmatrix}3&-2&-4\\4&-2&-5\\-1&1&2\end{bmatrix}=\frac{1}{2}\begin{bmatrix}0&6&3\\-6&0&6\\-3&-6&0\end{bmatrix}=\begin{bmatrix}0&3&\frac{3}{2}\\-3&0&3\\-\frac{3}{2}&-3&0\end{bmatrix}$ [1]

$Q'=\begin{bmatrix}0&-3&-\frac{3}{2}\\3&0&-3\\\frac{3}{2}&3&0\end{bmatrix}=-\begin{bmatrix}0&3&\frac{3}{2}\\-3&0&3\\-\frac{3}{2}&-3&0\end{bmatrix}=-Q$ Hence Q is a skew-symmetric matrix. [$\frac{1}{2}$]

$P+Q=\begin{bmatrix}3&1&-\frac{5}{2}\\1&-2&-2\\-\frac{5}{2}&-2&2\end{bmatrix}+\begin{bmatrix}0&3&\frac{3}{2}\\-3&0&3\\-\frac{3}{2}&-3&0\end{bmatrix}=\begin{bmatrix}3&4&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}=A$ [1]

Hence the matrix A is expressed as sum of a symmetric matrix and a skew symmetric matrix.]

14. HINTS : $\lim_{x\to a}\frac{x^2 f(a)-a^2 f(x)}{x-a}=\lim_{x\to a}\frac{x^2 f(a)-x^2 f(x)+x^2 f(x)-a^2 f(x)}{x-a}$ [$\frac{1}{2}$]

$=\lim_{x\to a}\frac{f(x)\left(x^2-a^2\right)-x^2\left[f(x)-f(a)\right]}{x-a}=\lim_{x\to a}\frac{f(x)\left(x^2-a^2\right)}{x-a}-\lim_{x\to a}\frac{x^2\left[f(x)-f(a)\right]}{x-a}$ [$\frac{1}{2}$]

$=\lim_{x\to a}\left[f(x)(x+a)\right]-\lim_{x\to a}x^2.\lim_{x\to a}\frac{f(x)-f(a)}{x-a}\quad\left[\because x\to a,\ so,\ x-a\neq 0\right]$ [1]

$$= \lim_{x\to a} f(x).\lim_{x\to a}(x+a) - \lim_{x\to a} x^2.\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = 2a.f(a) - a^2.f'(a)$$ [2]

$$\left[\because f(x) \text{ is differentiable at } x=a,\ so,\ \lim_{x\to a}\frac{f(x)-f(a)}{x-a} = f'(a)\right]]$$

OR, HINTS : $\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x} = k.\frac{1}{2}.\lim_{x\to\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)}{\frac{\pi}{2}-x}$ $\left[\because x\to\frac{\pi}{2}\ \therefore \frac{\pi}{2}-x\to 0\right]$ [2]

$= \frac{k}{2}.1 = \frac{k}{2}$ $\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$ Therefore, $\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}$ exists. [1]

Since f(x) is continuous at $x=\frac{\pi}{2}$, so $\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x} = f\left(\frac{\pi}{2}\right)$ $\Rightarrow \frac{k}{2}=3$ $\Rightarrow k=6$ [1]

15. HINTS : $y^{\frac{1}{n}} + y^{\frac{-1}{n}} = 2x$ … (i)

$\Rightarrow y^{\frac{1}{n}} - y^{\frac{-1}{n}} = \sqrt{\left(y^{\frac{1}{n}}+y^{\frac{-1}{n}}\right)^2 - 4.y^{\frac{1}{n}}.y^{\frac{-1}{n}}} = \sqrt{4x^2-4} = 2\sqrt{x^2-1}$ …(ii) [$\frac{1}{2}$]

$\Rightarrow y^{\frac{1}{n}} = \left(x+\sqrt{x^2-1}\right)$ [adding (i) & (ii)] $\Rightarrow y = \left(x+\sqrt{x^2-1}\right)^n$ [$\frac{1}{2}$]

$\frac{dy}{dx} = n\left(x+\sqrt{x^2-1}\right)^{n-1}\times\frac{d}{dx}\left(x+\sqrt{x^2-1}\right) = n\frac{\left(x+\sqrt{x^2-1}\right)^n}{x+\sqrt{x^2-1}}\times\left(1+\frac{1}{\not{2}}.\frac{1}{\sqrt{x^2-1}}\times\not{2}x\right)$ [2]

$\Rightarrow \frac{dy}{dx} = n\frac{y}{\left(x+\sqrt{x^2-1}\right)}\times\frac{\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2-1}}$ $\Rightarrow \left(x^2-1\right)\left(\frac{dy}{dx}\right)^2 = n^2y^2$ [1]

16. HINTS : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Differentiating both sides w. r. to x, we get, $\frac{2x}{a^2}-\frac{2y}{b^2}.\frac{dy}{dx}=0$ $\Rightarrow \frac{dy}{dx}=\frac{b^2}{a^2}\frac{x}{y}$ [$\frac{1}{2}$]

Slope of the tangent to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at (a secθ , b tanθ)

$=\left(\frac{dy}{dx}\right)_{at\ (a\sec\theta,\ b\tan\theta)} = \frac{b^2}{a^2}\frac{a\sec\theta}{b\tan\theta} = \frac{b}{a}.\operatorname{cosec}\theta\cdot$ [1]

$\therefore$ Slope of the normal to the curve at (a secθ , b tanθ) $= -\frac{1}{\left(\frac{dy}{dx}\right)_{at\ (a\sec\theta,\ b\tan\theta)}} = -\frac{a}{b}.\sin\theta\cdot$ [$\frac{1}{2}$]

Hence equation of the normal to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point (a secθ , b tanθ) is

$(y-b\tan\theta) = -\frac{a}{b}.\sin\theta(x-a\sec\theta) \quad \Rightarrow ax.\sin\theta + by = \left(a^2+b^2\right)\tan\theta$ [2]

OR, HINTS : $f(x)=\sin^3 x+\cos^3 x \quad \Rightarrow f'(x)=3\sin^2 x.\cos x+3\cos^2 x.(-\sin x)$ [1]

$\Rightarrow f'(x)=\frac{3}{\sqrt{2}}\sin 2x\times\sin\left(x-\frac{\pi}{4}\right) \quad \Rightarrow f'(0)=f'(\frac{\pi}{4})=f'(\frac{\pi}{2})=0$ [1]

In $\left(0,\frac{\pi}{4}\right)$, $\Rightarrow f'(x)<0$. So f(x) is strictly decreasing in $\left(0,\frac{\pi}{4}\right)$ [1]

In $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$, $\Rightarrow f'(x)>0$. So f(x) is strictly increasing in $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ [1]

17. HINTS: Let length of the edge of the cube = x units, radius of the sphere = r units.

Given, $6x^2 + 4\pi r^2 = k$ (k is a constant) - - - - - - - - - - (i) [$\frac{1}{2}$]

Let the total volume the cube and the sphere = V cubic units

Then $V = x^3+\frac{4}{3}\pi r^3$, - - - - - - - - - - - - -(ii) [$\frac{1}{2}$]

Differentiating both sides w. r. t x, we get, $\frac{dV}{dx}=3x^2+\frac{4}{\cancel{3}}\pi.\cancel{3}r^2.\frac{dr}{dx}=3x^2+4\pi r^2.\frac{dr}{dx}$. [$\frac{1}{2}$]

From (i), Differentiating both sides w. r. t x, we get, $\frac{dr}{dx}=-\frac{3x}{2\pi r}$. [$\frac{1}{2}$]

Therefore, $\frac{dV}{dx}=3x^2-6xr$.

For maximum or minimum value of V, $\frac{dV}{dx}=0 \Rightarrow$ x=2r (as $x\neq 0$) , [$\frac{1}{2}$]

Also, $\frac{d^2V}{dx^2}=6x-6r-6x.\frac{dr}{dx}=6x-6r-6x.\frac{-3x}{2\pi r}=12r-6r+\frac{9\times 4r}{\pi}=6r+\frac{9\times 4r}{\pi}>0$, [$\frac{1}{2}$]

i.e., $\frac{d^2V}{dx^2}=6r+\frac{9\times 4r}{\pi}>0$. Hence for x = 2r, volume V is minimum. $\frac{x}{2r}=\frac{1}{1}$. [1]

18. HINTS : $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx = \int \frac{\sin\{(x+\alpha)-2\alpha\}}{\sin(x+\alpha)}dx = \int \frac{\sin(x+\alpha).\cos 2\alpha - \cos(x+\alpha).\sin 2\alpha\}}{\sin(x+\alpha)}dx$ [1]

$= \cos 2\alpha \int dx - \sin 2\alpha \int \cot(x+\alpha)\ dx = \cos 2\alpha \int dx - \sin 2\alpha \int \cot(x+\alpha)\ d(x+\alpha)$ [2]

$= x.\cos 2\alpha - \sin 2\alpha.\log|\sin(x+\alpha)| + C$ [C is arbitrary constant of integration] [1]

19 HINTS : $(2x-10y^3)\frac{dy}{dx} + y = 0 \quad \Rightarrow \frac{(2x-10y^3)}{y} + \frac{dx}{dy} = 0 \quad \Rightarrow \frac{dx}{dy} + \frac{2}{y}x = 10y^2$ [1]

$\therefore$ Integrating factor (I.F) $= e^{\int \frac{2}{y}dy} = e^{2\int \frac{1}{y}dy} = e^{2\log y} = y^2$ [1]

$\therefore\ x.(I,F)e^{\int \frac{2}{y}dy} = \int 10y^2(I,F)\ dy \quad \Rightarrow x.y^2 = \int 10y^2.y^2\ dy$ [1]

$\Rightarrow x.y^2 = 10\frac{y^5}{5} + C$ [C is arbitrary constant of integration] $\Rightarrow x.y^2 = 2y^5 + C$ [1]

OR HINTS :

$y.\cos\frac{y}{x}(x\ dy - y\ dx) + x.\sin\frac{y}{x}(x\ dy + y\ dx) = 0$

$\Rightarrow dy(xy\cos\frac{y}{x} + x^2\sin\frac{y}{x}) - dx(y^2\cos\frac{y}{x} - xy\sin\frac{y}{x}) = 0$

$\Rightarrow \frac{dy}{dx}\left(\frac{y}{x}.\cos\frac{y}{x} + \sin\frac{y}{x}\right) = \left(\frac{y}{x}\right)^2\cos\frac{y}{x} - \frac{y}{x}\sin\frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2\cos\frac{y}{x} - \frac{y}{x}\sin\frac{y}{x}}{\frac{y}{x}.\cos\frac{y}{x} + \sin\frac{y}{x}} = f\left(\frac{y}{x}\right)$ [a homogenious differential equation] [1]

Let $\frac{y}{x} = x \quad \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \Rightarrow v + x\frac{dv}{dx} = \frac{v^2\cos v - v\sin v}{v.\cos v + \sin v}$ [$\frac{1}{2}$]

$\Rightarrow x\frac{dv}{dx} = \frac{v^2\cos v - v\sin v - v^2\cos v - v\sin v}{v.\cos v + \sin v}$

$\Rightarrow \frac{v.\cos v + \sin v}{2v\sin v}dv = -\frac{dx}{x} \quad \Rightarrow \frac{1}{2}\int\left(\cot v + \frac{1}{v}\right)dv = -\int\frac{dx}{x}$ [$\frac{1}{2}$]

$\Rightarrow \frac{1}{2}(\log|\sin v| + \log|v|) + \log|x| = \log|C|$ [1]

$\Rightarrow (\log|x^2 v\sin v|) = \log C^2 \quad \Rightarrow |xy\sin\frac{y}{x}| = C^2 \quad \Rightarrow \frac{\pi}{2} = C^2 \quad \left[\because f(1) = \frac{\pi}{2}\right]$ [$\frac{1}{2}$]

$\Rightarrow x^2y^2\sin^2\frac{y}{x} = \frac{\pi^2}{4}$ [$\frac{1}{2}$]

This is the particular solution of the given differential equation.

20. HINTS: The vertices of triangle ABC are given as A (1, 1, 2), B (2, 3, 5), and C (1, 5, 5).
The adjacent sides $\overrightarrow{AB}$ *and* $\overrightarrow{BC}$ of ΔABC are given as:

$$\overrightarrow{AB} = (1-3)\hat{i} + (-1+1)\hat{j} + (-3-2)\hat{k} = -2\hat{i} - 5\hat{k} \quad [\tfrac{1}{2}]$$

and $\overrightarrow{BC} = (4-1)\hat{i} + (-3+1)\hat{j} + (1+3)\hat{k} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ [$\frac{1}{2}$]

Area of ΔABC $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$ *sq units* [$\frac{1}{2}$]

$$Now,\ \left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \left|\begin{vmatrix} \hat{i} & j & \hat{k} \\ -2 & 0 & -5 \\ 3 & -2 & 4 \end{vmatrix}\right| = \left|-10\hat{i} - 7j + 4\hat{k}\right| = \sqrt{(-10)^2 + (-7)^2 + 4^2} \quad [2]$$

$$\therefore \text{ Area of } \Delta ABC = \frac{\sqrt{(-10)^2 + (-7)^2 + 4^2}}{2}\ sq\ units = \frac{\sqrt{165}}{2}\ sq\ units \quad [\tfrac{1}{2}]$$

21. HINTS: Let P be the point whose position vector is $\hat{i} + 3\hat{j} + 3\hat{k}$. Let the image of the point P be Q whose position vector is $a\hat{i} + b\hat{j} + c\hat{k}$. Let line segment PQ meets the given plane at R.

Then position vector of R is $\frac{a+1}{2}\hat{i} + \frac{b+3}{2}\hat{j} + \frac{c+3}{2}\hat{k}$. [$\frac{1}{2}$]

Vector equation of the st. line PR can be written as $\vec{r} = \left(\hat{i} + 3\hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

So general point on the st. line PR is $\vec{r} = (1+2\lambda)\hat{i} + (3-\lambda)\hat{j} + (2+\lambda)\hat{k}$ [$\frac{1}{2}$]

If this point lies on the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$

Then, (1+ 2 λ)2 + (3 – λ)(–1) + (2 + λ) + 3 = 0 $\Rightarrow \lambda = -\frac{2}{3}$ [1]

Hence vector of R is $\vec{r} = -\frac{1}{3}\hat{i} + \frac{11}{3}\hat{j} + \frac{4}{3}\hat{k}$ [$\frac{1}{2}$]

$$\therefore \frac{a+1}{2}\hat{i} + \frac{b+3}{2}\hat{j} + \frac{c+3}{2}\hat{k} = -\frac{1}{3}\hat{i} + \frac{11}{3}\hat{j} + \frac{4}{3}\hat{k}$$

$$\Rightarrow \frac{a+1}{2} = -\frac{1}{3},\ \frac{b+3}{2} = \frac{11}{3},\ \frac{c+3}{2} = \frac{4}{3} \qquad \Rightarrow a = -\frac{5}{3},\ b = \frac{13}{3},\ c = -\frac{1}{3} \quad [1]$$

Hence position vector of image (Q) is $a\hat{i} + b\hat{j} + c\hat{k}$ i.e., $\left(-\frac{5}{3}\hat{i} + \frac{13}{3}\hat{j} - \frac{1}{3}\hat{k}\right)$ [$\frac{1}{2}$]

22. HINTS: A : Cyclist, B : scooter driver, C : car drivers, E : Driver meets with an accident

Given, $P(A)=\frac{4000}{4000+8000+12000}=\frac{4}{24}$, $P(B)=\frac{8000}{24000}=\frac{8}{24}$, $P(C)=\frac{12000}{24000}=\frac{12}{24}$ [1]

$P(E/A)=0.02$, $P(E/B)=0.06$, $P(E/C)=0.30$ [1]

Using Bayes' theorem, required probability that the scooter driver meets with accident

$$=P(B/E)=\frac{P(B)P(E/B)}{P(A)P(E/A)+P(B)P(E/B)+P(C)P(E/C)}$$

$$=\frac{\frac{8}{24}\times 0.06}{\frac{4}{24}\times 0.02+\frac{8}{24}\times 0.06+\frac{12}{24}\times 0.30}=\frac{0.48}{0.08+0.48+3.60}=\frac{48}{416}=\frac{3}{26}$$ [2]

I would suggest students to use cycle as mode of transport as it is economical and environment friendly and keeps one physically fit.

23. HINTS: Here, X represents the number of sixes obtained when two dice are thrown simultaneously.

Therefore, X can take the value of 0, 1, or 2. [$\frac{1}{2}$]

P (X = 0) = P (not getting six on any of the dice) = $\frac{25}{36}$. [$\frac{1}{2}$]

P (X = 1) = P (six on first die and no six on second die) + P (no six on first die and six on second die) = $2\left(\frac{1}{6}.\frac{5}{6}\right)=\frac{10}{36}$. [$\frac{1}{2}$]

P (X = 2) = P (six on both the dice) = $\frac{1}{36}$ [$\frac{1}{2}$]

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of X = E(X) = $\sum X_i P(X_i)$ [$\frac{1}{2}$]

$$=0\times\frac{25}{36}+1\times\frac{10}{36}+2\times\frac{1}{36}=\frac{1}{3}$$ [$1\frac{1}{2}$]

SECTIONS – D (**Questions 24 to 29 carry 6 marks each**)

24. **HINTS:** It is given that f: **R** → {x ∈ **R**: $-1 < x < 1$} is defined as $f(x)=\frac{x}{1+|x|}$, $x\in R$.

Suppose f(x) = f(y), where x, y ∈ **R**. $\Rightarrow \dfrac{x}{1+|x|}=\dfrac{y}{1+|y|}$

It can be observed that if $x > 0$ and $y < 0$, then we have: $\dfrac{x}{1+x}=\dfrac{y}{1-y}$ $\Rightarrow 2xy = x - y$ [$\frac{1}{2}$]

Since x is positive and y is negative: $x > y \Rightarrow x - y > 0$

But, 2xy is negative. Then, $2xy \neq x - y$. [$\frac{1}{2}$]

Thus, the case of x being positive and y being negative can be ruled out.

Under a similar argument, x being negative and y being positive can also be ruled out [$\frac{1}{2}$]

∴ x and y have to be either both positive or both negative.
When x and y are both positive, we have:

$f(x)=f(y)$ $\Rightarrow \dfrac{x}{1+x}=\dfrac{y}{1+y}$ $\Rightarrow x+xy=y+xy$ $\Rightarrow x=y$ [$\frac{1}{2}$]

When x and y are both negative, we have:

$f(x)=f(y)$ $\Rightarrow \dfrac{x}{1-x}=\dfrac{y}{1-y}$ $\Rightarrow x-xy=y-xy$ $\Rightarrow x=y$ [$\frac{1}{2}$]

∴ f is one-one. [$\frac{1}{2}$]

Now, let y ∈ **R** such that $-1 < y < 1$.

If x is negative, then there exists $x=\dfrac{y}{1+y}\in R$ such that [$\frac{1}{2}$]

$$f(x)=f\left(\frac{y}{1+y}\right)=\frac{\frac{y}{1+y}}{1+\left|\frac{y}{1+y}\right|}=\frac{\frac{y}{1+y}}{1+\frac{-y}{1+y}}=y$$ [1]

If x is positive, then there exists $x=\dfrac{y}{1+y}\in R$ such that [$\frac{1}{2}$]

$$f(x)=f\left(\frac{y}{1-y}\right)=\frac{\frac{y}{1-y}}{1+\left|\frac{y}{1-y}\right|}=\frac{\frac{y}{1-y}}{1+\frac{y}{1-y}}=y$$ [$\frac{1}{2}$]

∴ f is onto. [$\frac{1}{2}$]

Hence, f is one-one and onto.]

OR, **HINTS:** Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation * on X is defined as: $a*b=\begin{cases} a+b, & if\ a+b<6 \\ a+b-6, & if\ a+b\geq 6 \end{cases}$

An element $e \in X$ is the identity element for the operation *, if $a*e=a=e*a \quad \forall a\in X$. [1]

For $a\in X$, we observed that :

$a*0=a+0=a \quad [a\in X \quad \Rightarrow a+0<6] \quad \& \quad 0*a=0+a=a \quad [a\in X \quad \Rightarrow 0+a<6]$ [1]

$a*0=a=0*a \quad \forall a\in X$

Thus, 0 is the identity element for the given operation *. $[\frac{1}{2}]$

An element $a \in X$ is invertible if there exists $b\in X$ such that $a * b = 0 = b * a$. [1]

i.e., $\begin{cases} a+b=0=b+a, & if\ a+b<6 \\ a+b-6=0=b+a-6, & if\ a+b\geq 6 \end{cases}$ [1]

i.e., $a = -b$ or $b = 6 - a$ $[\frac{1}{2}]$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$. $[\frac{1}{2}]$

$\therefore b = 6 - a$ is the inverse of a " $a \in X$. $[\frac{1}{2}]$

Hence, the inverse of an element $a \in X$, $a \neq 0$ is $6 - a$ i.e., $a^{-1} = 6 - a$.

25. HINTS: $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2(b+c)^2 \\ b^2 & (c+a)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix}$ $\begin{matrix}(C_2' \to C_2 - C_1) \\ (C_3' \to C_3 - C_1)\end{matrix}$ [1]

$= (a+b+c)^2 \begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$ (Taking common from $C_2 \,\&\, C_3$) [1]

$= (a+b+c)^2 \begin{vmatrix} 2bc & -2c & -2b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$ $[R_1' \to R_1' - (R_2 + R_3)]$ [1]

$$= 2(a+b+c)^2 \begin{vmatrix} bc & 0 & 0 \\ b^2 & c+a & \frac{b^2}{c} \\ c^2 & \frac{c^2}{b} & a+b \end{vmatrix} \quad \begin{matrix} C_2' \to C_2 + \frac{1}{b}C_1 \\ C_3' \to C_3 + \frac{1}{c}C_1 \end{matrix}$$ [1]

$$= 2bc(a+b+c)^2(ca+\not{cb}+a^2+ab-\not{bc}) = 2abc(a+b+c)^3 = \text{RHS}$$ [2]

OR, **HINTS** : Since a, b, c, are in A.P, so a+c = 2b [1]

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = \begin{vmatrix} 2x+6 & 2x+8 & 2x+2(a+c) \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \quad [R_1' = R_1 + R_3]$$ [2]

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+4b \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \quad [\because a+c=2b]$$ [1]

$$= \begin{vmatrix} 2x+6-2(x+3) & 2x+8-2(x+4) & 2x+4b-2(x+2b) \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \quad [R_1' = R_1 - 2R_2]$$ [1]

$$= \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$$ [1]

26. **HINTS** : Let $R = \{(x, y) : x^2 \le y \le |x|\}$ $\Rightarrow R = \{(x, y) : x^2 \le y\} \cap \{(x, y) : y \le |x|\}$ [$\frac{1}{2}$]

$\Rightarrow R = \{(x, y) : x^2 \le y\} \cap \{\{(x, y) : y \le x,\ x \ge 0\} \cup \{(x, y) : y \le -x,\ x < 0\}\}$ [$\frac{1}{2}$]

$\Rightarrow R = R_1 \cap (R_2 \cup R_3)$ $\Rightarrow R = (R_1 \cap R_2) \cup (R_1 \cap R_3)$

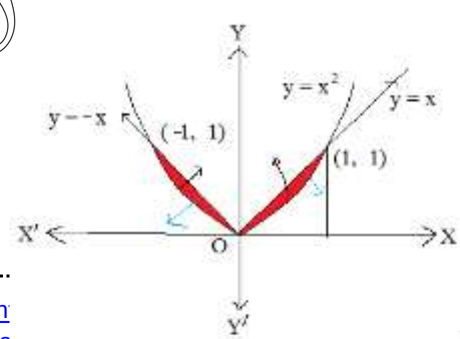
$R_1 = \{(x, y) : x^2 \le y\}$ = The interior region of the parabola $x^2 = y$, whose opening is towards positive direction of Y – axis [$\frac{1}{2}$]

$R_2 = \{(x, y) : y \le x,\ x \ge 0\}$

$\therefore R_2$ is the region lying below the st. line $y = x$ [$\frac{1}{2}$]

$R_3 = \{(x, y) : y \le -x,\ x < 0\}$

$\therefore R_3$ is the region lying below the st. line $y = -x$ [$\frac{1}{2}$]



[1]

$\left(R_1 \cap R_2\right)$ = area of shaded region in 1st quadrant

$\left(R_1 \cap R_3\right)$ = area of shaded region in 2nd quadrant $[\frac{1}{2}]$

Since the parabola $x^2 = y$ & $y = |x|$ are symmetrical about Y-axis, so areas of the shaded region are equal numerically.

Hence the required area $= \left(R_1 \cap R_2\right) \cup \left(R_1 \cap R_3\right)$

= 2 | area of one shaded region | (say area in 1st quadrant)

$$= 2\left|\int_0^1 (x - x^2)\, dx\right| = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\left(\frac{1}{2} - \frac{1}{3}\right) - 0\right] = \frac{1}{3} \text{ sq. units}$$ [2]]

27. **HINTS**: LHS= $\int_0^{\frac{\pi}{2}} \frac{\sin x.\cos x}{a^2.\cos^2 x + b^2.\sin^2 x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin x.\cos x}{\left(a^2 - b^2\right).\cos^2 x + b^2} dx$ $[\because a > 0,\ b > 0,\ a \neq b]$ [1]

$$= \frac{-1}{2\left(a^2 - b^2\right)} \int_0^{\frac{\pi}{2}} \frac{(-2\sin x.\cos x)}{\cos^2 x + \frac{b^2}{a^2 - b^2}} dx = \frac{-1}{2\left(a^2 - b^2\right)} \int_0^{\frac{\pi}{2}} \frac{d\left(\cos^2 x + \frac{b^2}{a^2 - b^2}\right)}{\cos^2 x + \frac{b^2}{a^2 - b^2}}$$ $[\because a \neq b]$ [2]

$$= \frac{-1}{2\left(a^2 - b^2\right)}\left[\log\left|\left(\cos^2 x + \frac{b^2}{a^2 - b^2}\right)\right|\right]_0^{\frac{\pi}{2}} = \frac{-1}{2\left(a^2 - b^2\right)}\left[\log\left|\left(0 + \frac{b^2}{a^2 - b^2}\right)\right| - \log\left|\left(1 + \frac{b^2}{a^2 - b^2}\right)\right|\right]$$ [2]

$$= \frac{-1}{2\left(a^2 - b^2\right)}\left[\log\left(\frac{\frac{b^2}{a^2 - b^2}}{\frac{a^2}{a^2 - b^2}}\right)\right] = \frac{1}{\not{2}\left(a^2 - b^2\right)}\left[\not{2}\log\left(\frac{a}{b}\right)\right]$$ $[\because a > 0,\ b > 0,\ a \neq b]$ $= \frac{\log\left(\frac{a}{b}\right)}{a^2 - b^2}$ [1]

OR, **HINTS** : By definition, $\int_a^b f(x)\, dx = (b - a) \lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f(a + rh)$, *where*, $nh = b - a$ [1]

Here, a = 1, b = 2, f(x) =2x + 5, nh=2 – 1 = 1

$\therefore \int_1^2 (2x + 5)\, dx = 1. \lim_{h \to 0} h \sum_{r=1}^{n} [2(1 + rh) + 5]$, $nh = 1$ $\because n \to \infty, \therefore h \to 0$ [1]

Now, $h\sum_{r=1}^{n}\left[2(1+rh)+5\right]=2h^2\sum_{r=1}^{n}(r)+h\sum_{r=1}^{n}7$ [1]

$=\not{2}h^2.\frac{n(n+1)}{\not{2}}+7nh=nh(nh+h)+7nh=1(1+h)+7=h+8$ [2]

$\therefore\int_1^2(2x+5)\ dx=\lim_{h\to 0}(h+8)=\lim_{h\to 0}h+8=8$ [1]

28. **HINTS** : Given a l + bm + cn = 0 $\Rightarrow n=-\frac{al+bm}{c}$.

Substituting $n=-\frac{al+bm}{c}$ in the equation fmn + gnl + hlm = 0, we get, $[\frac{1}{2}]$

$-fm\frac{al+bm}{c}-gl\frac{al+bm}{c}+hlm=0 \qquad \Rightarrow -afml-bfm^2-agl^2-bgml+chlm=0$

$\Rightarrow agl^2+bfm^2+(af+bg-ch)ml=0 \qquad \Rightarrow ag\left(\frac{l}{m}\right)^2+(af+bg-ch)\frac{l}{m}+bf=0 \quad \cdots(i)$ $[1\frac{1}{2}]$

This is a quadratic equation in $\frac{l}{m}$.

Let the D.C's of two st. lines be l_1, m_1, n_1 and l_2, m_2, n_2 [1]

Now from (i), $\frac{l_1}{m_1}.\frac{l_2}{m_2}=\frac{bf}{ag} \qquad \Rightarrow \frac{l_1l_2}{\frac{f}{a}}=\frac{m_1m_2}{\frac{g}{b}}=k\ (let) \quad \cdots(ii)$ [k ≠ 0] [1]

Similarly, from the first two given equations eliminating m and proceeding above, we get,

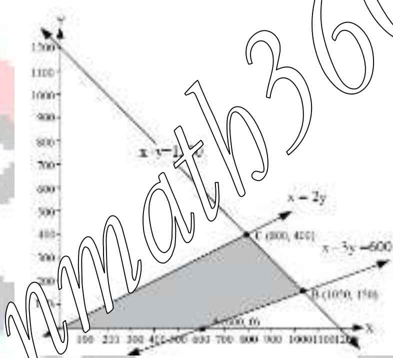
$\frac{l_1l_2}{\frac{f}{a}}=\frac{n_1n_2}{\frac{h}{c}}=k \quad \cdots(ii)$ [1]

Since the lines are perpendicular, so $l_1l_2+m_1m_2+n_1n_2=0 \quad \Rightarrow \frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$. [∵ k ≠ 0] [1]

29. HINTS : Let x and y be the number of dolls of type A and B respectively that are produced per week.
The given problem can be formulated as follows: Maximize $z = 12x + 16y$ … (1)
subject to the constraints,
$x+y\le 1200 \quad \cdots(2) \qquad x\ge 2y \quad \cdots(3) \qquad x-3y\le 600 \quad \cdots(4) \qquad x,\ y\ge 0 \quad \cdots(5)$ [2]
The feasible region determined by the system of constraints is as follows.
The corner points are A (600, 0), B (1050, 150), and
C (800, 400).

The values of z at these corner points are as follows.

Corner point	$z = 12x + 16y$	
A (600, 0)	7200	
B (1050, 150)	15000	
C (800, 400)	16000	→ Max [1]



The maximum value of z is 16000 at (800, 400). [graph 2]

Thus, 800 and 400 dolls of type A and type B should be produced respectively to get the maximum profit of Rs 16000. [1]

"*The only way to learn MATHEMATICS is to do MATHEMATICS.*" – Paul Halmos.

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