CLASS XII
SAMPLE PAPER
MATHS

Time Allowed : 3 hours **Maximum Marks : 100**

General Instructions : –

- **All questions are compulsory.**
- **The question paper consists of 29 questions divided in to three sections A, B and C. Section A comprises of 10 questions of 1 mark each, Section B comprises of 12 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.**
- **All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.**
- **There is no over all choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.**
- **Use of calculators is not permitted**

Section – A (Q1- Q10 each carry one mark.)

1. Evaluate : $\int \frac{Sin\sqrt{x}\ dx}{\sqrt{x}}$

2. Find the value of x , if

$$\begin{pmatrix} 3x+y & -y \\ 2y-x & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -5 & 3 \end{pmatrix}$$

3. Find the principle value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

4. Find the value of p if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+3\hat{j}+p\hat{k})=\vec{0}$.

5. Write the direction cosines of a line equally inclined to the three coordinate axes.

6. If $\vec{p}$ is a unit vector and $(\vec{x} - \vec{p}) \cdot (\vec{x} + \vec{p}) = 80$, then find $|\vec{x}|$.

7. Find the value of x if

$$\begin{vmatrix} x & -4 \\ 8 & -2x \end{vmatrix} = 0$$

8. Write the value of the following determinant :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

9. Find $\vec{a} \cdot \vec{c}$, if $|\vec{a}| = 5$, $|\vec{c}| = 4$ and $\vec{a} \times \vec{c} = 8$.

10. Find $\frac{dy}{dx}$, *if* $y = e^{ax}$,

Section – B (Q11- Q22 each carry 4- marks.)

11. If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

OR

If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

12. Evaluate :

$$\int x \sin^{-1} x \, dx$$

OR

$$\int_0^{\frac{\pi}{2}} \sqrt{\tan x} + \sqrt{\cot x}\ dx.$$

13. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

14. Evaluate:

$$\int \frac{2x^2 + 3 \quad dx\,.}{(x^2 - 1)(x^2 + 4)}$$

15. Find the shortest distance between the following two lines :

$$\vec{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k};$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

16. Prove the following :

$$\cot^{-1}\left(\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right) = \frac{x}{2}, \quad x \in \left(0, \frac{\pi}{4}\right)$$

OR

Solve for x :

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\ \text{cosec}\ x)$$

17. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

18. Find the equation of the tangent and normal to the curve $x= a\sin^3 t$, $y= b\cos^3 t$ at the point $t= \pi/4$.

OR

If $x= a(\cos t + t\sin t)$, $y= b(\sin t - t\cos t)$. find $\frac{d^2y}{dx^2}$,

19. Prove that : $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$.

20. Solve the following differential equation :

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

21. Solve the following differential equation :

$(1+y^2)dx = (\tan^{-1} y - x)\, dy$, given $y(0) = 0$

22. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that

$$(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$$

Section – C (Q23- Q29 each carry 6- marks.)

23. Obtain the inverse of the following matrix using elementary operations :

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

OR

Find the inverse of the $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and hence solve the system of equations:

x +2y+z= 4 , -x+y+z= 0 , x- 3y+z = 2

24. Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z + 8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 & 2x + y – z + 5 = 0

25. Draw the rough sketch of {(x,y) : $|x-1| \leq y \leq \sqrt{5-x^2}$ }.Using Integration find the area of the region enclosed.

26. Evaluate:

$$\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

27. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle 30^0 is $\frac{4}{81}\pi h^3$.

OR

A manufacturer can sell x items at a price of Rs. $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit.

28. A farmer has a supply of chemical fertilizer of type I which contains 10% Nitrogen and 5% Phosphoric acid and type II which contains 6% Nitrogen and 10% Phosphoric acid. After soil testing it was found that at least 14kg of Nitrogen and 14kg of Phosphoric acid is required for a good crop. The type I costs Rs. 2/kg and the type II cost Rs. 3/kg. How

many kilograms of each fertilizer should be used to meet the requirement so that the cost be minimum?

29. A lot of 100 bulbs is known to contain 10 defective and 90 non-defective bulbs. If 8 bulbs are selected at random, what is the probability that

(i) there will be 3 defective and 5 non-defective bulbs

(ii) there will be atleast one defective bulb.