AIMS Test Series-2011

M.M.: 100 Class –XII **By: SandeepShishodia**
Time: 3 Hours **Code: AIMS12I11** *M.Tech. PGT(Maths)*

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted. You may ask for logarithmic tables, if required.*

Section – A

Questions number 1 to 10 carry 1 marks each:

1. If $A=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$, then find A^2 hence find A^6.
2. For what value of a, $\begin{pmatrix}2a&-1\\-8&3\end{pmatrix}$ is a singular matrix.
2. Write the number of all one-one functions from the set A = {a, b, c} to itself.
3. Find $f(x)$ satisfying the following: $\int e^x\left(\sec^2 x+\tan x\right)dx=e^x f(x)+c.$
4. Find the value of λ such that the line $\frac{x-2}{9}=\frac{y-1}{\lambda}=\frac{z+3}{-6}$ is perpendicular to the plane 3x-y-2z = 7.
5 Evaluate $\int_0^1\left[\{x\}\right]dx$ .Where {.} is fractional part and [.] is greatest integer function.
6. Given $\overrightarrow{AB}=3\hat{i}-\hat{j}-\hat{k}$ and coordinate of the terminal point are (0, 1, 3). Find the coordinates of the initial point.
7. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on vector $7\hat{i}-\hat{j}+8\hat{k}$.
8. If $f'(x)=\frac{1}{x}+x$ and $f(1)=\frac{5}{2}$, then what is $f(x)$?

9. Find the value of $arc\ \sin\left(\sin\frac{2\pi}{3}\right)$.

10. Without expanding the determinant show that: $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix} = 0$.

Section-B

Questions number 11 to 22 carry 4 marks each.

11. From a pack of 52 playing cards, a card is accidently dropped. From the remaining 51 cards two cards are drawn at random (without replacement) and are found to be both spades. Find the probability that the dropped card was a club?

12. Find whether the lines $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect or not. If they intersect, find the point of intersection. If do not intersect, find shortest distance between them.

OR

Prove that if a plane has the intersepts a, b, c and is at a distance of p units from the origin, then $a^{-2} + b^{-2} + c^{-2} = p^{-2}$

13. $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors. Suppose $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

OR

If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

14. Evaluate: $\int \frac{1}{x^4 + 1} dx$.

15. Let $A = N \times N$. Let * be binary operation on A defined by $(a,b) * (c,d) = (ad + bc, bd)$. Then
    (i) find the identity element of $(A, *)$
    (ii) find the invertible elements of $(A, *)$.

16. Solve for x, $2\tan^{-1}(\cos x) = \tan^{-1}(2\cos ec\ x)$.

OR

Prove that: $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan^{-1}\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$.

17. Prove by using properties of determinants:

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3.$$

18. For what value of a and b, the function defined as:

$$f(x)=\begin{cases} 3ax+b & ;\ if \quad x<1 \\ 11 & ;\ if \quad x=1 \\ 5ax-2b & ;\ if \quad x>1 \end{cases}$$ is continuous at $x=1$.

19. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\dfrac{\left[1+(y')^2\right]^{3/2}}{y''}$ is a constant and free from a and b.

OR

If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\dfrac{dy}{dx}=\sqrt{\dfrac{1-y^2}{1-x^2}}$.

(2)

20. Find the interval in which the function f given by $f(x)=\sin^4 x+\cos^4 x$ is strictly increasing or strictly decreasing.

21. Solve the differential equation: $\dfrac{dy}{dx}-\dfrac{1}{x}.y=2x^2$.

22. Solve: $(x^3+x^2+x+1)\dfrac{dy}{dx}=2x^2+x$.

Section-C

Question number 23 to 29 carry 6 marks each:

23. Using integration find the area of the region included between the parabola $y^2=x$ and the line $x+y=2$.

OR

Using definite integration, find the area of the region: $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$.

24. A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that all the balls are blue?

25. Find the equation of the line of intersection of planes
$4x+4y-5z=12$ and $8x+12y-13z=32$ in the vector and symmetric form.

26. A window is in the shape of a rectangle surmounted by a semicircle. If its perimeter is 30 m, then find the dimensions of the window so that it may admit maximum light.

27. Evaluate $\int_1^4 (x^2 - x)dx$ as the limit of a sum.

28. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$ find A^{-1}, and hence solve the following system of equations:

$$2x + y + 3z = 3$$
$$4x - y = 3$$
$$-7x + 2y + z = 2.$$

OR

Obtain the inverse of the following matrix using elementary transformations: $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$.

29. There are two factories located one at place Vidyut Nagar and other at Delhi, from these locations, a certain number of machines is to be delivered to each of the three depots situated at P, Q and R. The weekly requirements of the depots are respectively 5, 5, and 4 units of the machines while the production capacity of the factories at Vidyut Nagar and Delhi are 8 and 6 units respectively. The cost of transportation per unit is given below.

From / To	Cost (in Rs.)		
	P	Q	R
Vidyut Nagar	160	100	150
Delhi	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum?
What is the minimum transportation cost?

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