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Roll.No:

SAMPLE PAPER (2011)

MATHEMATICS [XII]

No of Printed Pages:4

Maximum Marks :100

INSTRUCTIONS:

1. There are twenty nine questions in this paper divided into three sections-: A, B and C.
2. Section A contains 10 questions of ONE mark each; Section B contains 12 questions of FOUR marks each; and Section C has 7 questions of SIX marks each.
3. Attempt all questions.
4. Rough work, if any, should be shown in the right hand margin(of approx. 2 inches) in the same page where the respective question has been solved.

SECTION- A

1. Find x and y , if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$
2. Find the angle between two vectors \vec{a} and \vec{b} with magnitude $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.
3. Check whether the relation R in the set $\{1,2,3\}$ given by $R = \{(1,2), (2,1)\}$ is transitive.
4. Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.
5. Determine the direction cosines of the normal to the plane
 $2x + 3y - z = 5$

6. Find the integral $\int_{0.2}^{3.5} [x] dx$

7. Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

8. How many orders are possible for a matrix having 8 elements ?

9. Evaluate $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$

10. Find x , if

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

SECTION-B

11. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\text{and } \hat{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

12. If x,y,z are all different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $xyz = -1$

13. Show that a function

$$f : N \rightarrow N, \text{ given by } f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases} \text{ is bijective.}$$

14. Solve for x :: $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$

15. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one . Find the value of

λ .

OR

If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, and
 $|\vec{a}| = 3$, $|\vec{b}| = 4$, and $|\vec{c}| = 5$. Find
the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

16. Find the intervals in which the following function is increasing

$$f(x) = 2x^3 - 15x^2 + 36x + 17$$

17. Evaluate $\int_0^{\frac{3}{2}} |x \cos \pi x| dx$

18. Form the differential equation representing the family of parabolas having vertex at the origin and the axis along +ve x-axis.

19. Solve the following differential equation

$$\cos^2 x \frac{dy}{dx} + y = \tan x.$$

OR

$$x \frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

20. An urn contains 5 white and 3 red balls. Find the probability distribution of the number of red balls, with replacements, in three draws.

21. If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, Find $\frac{dy}{dx}$

OR

If $y = e^{\cos^{-1} x}$, $-1 \leq x \leq 1$ show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$

22. Determine the values of a, b, c if the function

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & \text{if } x < 0 \\ c & \text{if } x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & \text{if } x > 0 \end{cases} \quad \text{is continuous at } x = 0$$

OR

Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points. Here $[x]$ denotes the greatest integer less than or equal to x .

SECTION-C

23. Draw a rough sketch indicating the following region and find its area:

$$\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$$

OR

$$\{(x, y): y^2 \leq 3x, 3x^2 + 3y^2 \leq 16\}$$

24. Prove that the semi-vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1} \frac{1}{3}$.

25 . Show that the lines $\frac{x - a + d}{\alpha - \delta} = \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta}$
and $\frac{x - b + c}{\beta - \gamma} = \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \gamma}$ are co-planer.

26. In answering a question on a multiple choice test a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with the probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly.

27. Evaluate the following integral as the limit of a sum : $\int_0^2 (x^2 + x + 2) dx$

28. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs. 7 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates the machine for atmost 12 hours a day.

29. Solve the following system of equation by matrix method:

$$3x - 2y + 4z = 2$$

$$2y - 3z = 1$$

$$x - y + 2z = 1$$

OR

If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$
