CLASS XII
MATHS
DIFFERENTIAL CALCULUS

SECTION-A

1. Differentiate $e^{\log(\tan x)}$ w.r.t. log (secx).

2. Discuss the applicability of LMVT for f(x) = $x^{\frac{2}{3}}$ on [2,3]

3. Find the maximum and minimum values, if any of $f(x) = -|x+2|+3$

4. Find 'a' for which f(x) = $x^4 - 62x^2 + ax + 9$ attains its maximum value at x=1, in the interval [0,2].

5. At what point on curve $x^2 + y^2 - 2x - 4y + 1 = 0$, is the tangent is parallel to the y-axis.

6. If radius of a circle is increased from 5 cm to 5.1 cm, find the approximate increase in it's area.

7. Find the interval in which the function e^{x^2} is monotonic.

8. The cost function of a firm is given by C=$4x^2$-x+70. Find the marginal cost, when x=3.

9. Differentiate w.r.t.x: $x^e + 5^{\log(\sin)x}$

10. Show that $9x^3$+4 x +5 is always increasing.

11. Differentiate x^2 w.r.t. $\log_{10}$ x

12. Find the point on the curve y^2 = 8x for which the abscissa and ordinate change at the same rate.

13. Find the maximum and minimum values, if any of $f(x) = |\cos 2x| + 3$

14. Find 'a' for which f(x) = a(x+sinx)+a is increasing .

15. For the curve y = $3x^2$ + 4x, find the slope of the tangent to the curve at the point x = -2.

16. Find a point on the parabola y =$(x – 3)^2$,where the tangent is parallel to the chord joining (3, 0) and (4,1).

17. Check the monotonocity i.e increasing & decreasing of $f(x) = \cos 2x, [\pi/2, \pi]$.

18. If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then find dy/dx.

19. Differentiate w.r.t.x: $x^e + a^x$

20. Show that the function log(cosx) is always decreasing.

SECTION-B

1. If $y = x^{\cos x} + (\cos x)^x$, find $\frac{dy}{dx}$

2. The volume and surface area of a variable sphere are changing at the of 200 cm^3/sec and 10 cm^2/sec respectively. Find the rate of change of its radius at that moment.

3. Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles if $k^2 = 8$.

4. Find the intervals on which function -- $2x^3 – 9x^2 - 12x + 1$ is strictly increasing and decreasing.

5. If $y = e^x \tan^{-1} x, show that (1+x^2)\frac{d^2y}{d^2x} - 2(1-x+x^2)\frac{dy}{dx} + (1-x)^2 y = 0$

6. If $y = e^{a\cos^{-1}x}, show that (1-x^2)\frac{d^2y}{d^2x} - x\frac{dy}{dx} - a^2 y = 0$

7. Let f(x) = $\begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if} \quad x < \frac{\pi}{2} \\ a & \text{if} \quad x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi - 2x)^2}, & \text{if} \quad x > \frac{\pi}{2} \end{cases}$ be a continuous function at x = $\frac{\pi}{2}$, find a and b.

8. *If* $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & , x < 0 \\ c & , x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{b\sqrt{x^3}} & , x > 0 \end{cases}$ is continuous at x=0. Find values of a, b, c.

9. If $y = (\tan^{-1}x)^2$, prove that $(1 + x^2)y_2 + 2x(1+x^2)y_1 = 2$

10 If $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$ and y=asint find $\frac{d^2y}{dx^2}$

11. Find the interval in which the function given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x}$ is increasing or decreasing.

12. The tangent to the curve $y=ax^3 + bx^2 + cx + 5$ touches the x-axis at the point (-2,0) and cuts the y-axis atthe point where its gradient is 3.Find the values of a, b, and c.

13. If x = a sin2t (1+cos2t) and y = b cos2t(1- cos2t),find $\frac{dy}{dx}$ at x = $\frac{\pi}{4}$

14. If y = $\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$ find $\frac{dy}{dx}$.

15. If sin x = y sin (x+b), show that $\frac{dx}{dy} = \frac{\sin^2(x+b)}{\sin b}$

16. Prove that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\left(\frac{x}{a}\right) + \left(\frac{y}{b}\right) = 2$ at (a,b)for all n$\in N$

17. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

18. Water is running into a conical vessel,15cm deep and 5cm in radius, at the rate of 0.1cm^3/ sec. When the water is 6 cm deep, find at what rate is (i) the water level rising? and (ii) the water surface are increasing?

19. Find the intervals on which function $x^4 - 2x^2$ is strictly increasing and decreasing.

20. Find the value of 'k' so that the function f is continuous at the indicated point.

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{if } x \neq \dfrac{\pi}{2}, \\ 3, & \text{if } x = \dfrac{\pi}{2} \end{cases} \quad \text{at } x = \frac{\pi}{2}$$

21.
Find the value of a and b such that the function defind by

$$f(x) = \begin{cases} x^2 + ax + b & \text{if } 0 \le x < 2 \\ 3x+2 & \text{if } 2 \le x \le 4 \\ 2ax + 5b & \text{if } 4 < x \le 8 \end{cases} \quad \textit{is a continous function on } [0,8].$$

22. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

23. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ for -1<x <1 show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

24. Find a point on the parabola $y^2 = 4x$ which is nearest to the point (2,-8).

25. A ladder 5m long is leaning against a wall . The bottom of the ladder is pulled along the ground, away from the wall at the rate of 2 m./sec.How fast is its height decreasing on the wall, when the foot of the ladder is 4 m away from the wall.

26. If y = $[\log(x + \sqrt{1+x^2})]^2$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 2 = 0.$

27. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4,2]$.

28. If $f(x) = 3x^2 + 15x + 5$, then find the approximate value of $f(3.02)$, using diffrentials.

29. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ w.r.t. $Sin^{-1}\left(\frac{2x}{1+x^2}\right)$

SECTION-C

1. An open box with a square base is to be made out of a given quantity of sheet of area a^2. Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.

2. Show that the altitude of a right circular cone of least curved surface area and given volume equal to $\sqrt{2}$ times the radius of the base.

3. A rectangle is inscribed in a semicircle of radius 'a' with one of its sides on the diameter of semicircle Find the dimension of the rectangle so that its area is maximum . An isosceles triangle is inscribed in an ellipse with one vertex coinciding with the end of its major axis. Find the maximum area of the triangle.

4. A rectangular window is surmounted by an equilateral triangle. If the perimeter is 16m, find the width of the window so that maximum amount of light may enter.

5. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2\alpha$

6. Show that the height of the cylinder of maximum volume which can be inscribed in a given cone, is one-third of the height of the cone.

7. A rectangular sheet of tin of side 45cm x 24cm is made into a open box by cutting off squares from each corner and folding up the flaps. What should be the side of the square to be cut off from each corner so that the volume of the box is maximum.

8. Show that altitude a of right circular cone of maximum volume that can be inscribed in a sphere of radius r.

9. A rectangle is inscribed in a semicircle of radius 'a' with one of its sides on the diameter of semicircle. Find the dimension of the rectangle so that its area is maximum. Find also the area.

10. Show that the semi-vertical angle of a right circular cone of given surface and maximum volume is $\sin^{-1} 1/3$.

11. Prove that the Volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

12. A given quantity of metal is to be cast into a solid half circular cylinder(i.e. with rectangular base and semicircular ends). Show that in order that the surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : \pi + 2$.

13. A wire of length 28m is cut into two pieces. One is made into a square and the other in to a circle. How should the wire be cut so that their combined area is minimum.

14. A square sheet of tin of side 18 cm is made into an open box by cutting equal squares from the four corners and folding up the flaps. What should be the side of the square to be cut off from each corner so that the volume of the box is maximum.

15. Prove that the surface area solid cuboid of square base and given volume, is minimum when it is a cube.