**Subject : Mathematics**

**Time : 3 Hours 15 Minutes** **Full Marks : 100**

- **Special Credit will be given for answers which are brief and to the point**
- **Marks will be deducted for spelling mistakes, untidiness and bad handwriting**
- **Figures in the margin indicate full marks for the questions**

## Group – A

1. **Answer the following questions (Alternatives are to be noted) : 1 X 10 = 10**

(a) If f(x) is a polynomial of degree n(≥1), then the degree of f'(x) is (n+1)

(b) The sum of the coefficients in the expansion of $(1+X)^{12}$ is
(i) 4096 (ii) 2048 (iii) 1024 (iv) 512

(c) The eccentricity of the parabola $x^2+y-x+5=0$ is
(i) >1 (ii) <1 (iii) 0 (iv) 1

0r If y = (x-1)log(2x+1), then the value of dy/dx at the point x=1 is ---------

(d) The point which is exterior to the ellipse $16x^2+9y^2-16x-32=0$ is
(i) (1/3,1) (ii) (3,-2) (iii) (1/4,1) (iv) (½,2)

(e) Fill in the blank :
If y = 1+cos2x, then $(d^2y/dx^2) + 4y =$

Or $\int(x+1)/(\sqrt{x} + 1)\,dx$ is ----------------------------------

(f) Define collectively exhaustive events in relation to probability theory

(g) Give the classical definition of probability

(h) If any point on a hyperbola is (4 secφ, 5 tanφ), then the eccentricity of the Hyperbola is
(i) 25/16 (ii) √41/4 (iii) √41/5 (iv) √41/8

(i) If f(x) =IxI, then f(x) is
(i) differentiable at x=0 (ii) not differentiable at x=0
(iii) Lf′(0) = Rf′(0) (iv) none of these

Or Two cards are drawn from a well shuffled pack of 52 cards. The probability of getting ace or black card is
(i) 17/52 (ii) 55/221 (iii) 4/13 (iv) none of these

(j) If X=1/3 +1.3/3.6+1.3.5/3.6.9+……. Then the value of $x^2+2x$ is :
(i) $(1/3)^1/3$ (ii) √3/2 (iii) √1/3 (iv) $(3/2)^1/3$

## Group – B

2. **Answer the following questions as per instruction :**

**(a) Answer any two questions : 2 x 2 = 4**

(i) Find the probability of obtaining six atleast once in four throws of an Unbiased die.

(ii) Using the series for e show that 2<e<3

(iii) If the length of the minor axis of an ellipse is twice the length of its latus Rectum, then what is the eccentricity of the ellipse?

**(b) Answer any one question : 2 X 1 = 2**

(i) If the coordinates of the two foci of an ellipse are (0,4) & (0,-4) and the Equations of the directrices are y=9 & y=-9, then find the equation of the Ellipse and the length of its latus rectum.

(ii) Find the coordinates of two points on the parabola $x^2=8y$ such that the distance of each point from the focus is 4 units. Find the equation of the circle described on the line segment joining these points as a diameter

**( C) Answer any one question :** **$2 \times 1 = 2$**

(i) Find $dy/dx$ when $xy+1=\cos(xy)$

(ii) If $dx/dy=u$ and $d^2x/dy^2=v$, show that $d^2y/dx^2=-v/u^3$

**(d) Evaluate any one of the following integrals :** **$2 \times 1 = 2$**

(i) $\int \cos x/\sqrt{1+\sin x}\ dx$ (ii) $\int \sqrt{\tan x}/\sin x \cos x\ dx$

**(e) Answer any one question :** **$2 \text{ X } 1 = 2$**

(i) Find the equation of the ellipse whose major axis and minor axis are along x-axis and y-axis respectively, eccentricity is $1/\sqrt{2}$ and sum of the squares of lengths of the major and minor axes is 24.

(ii) If the derivatives of two functions are equal, then prove that the difference of the two functions is constant.

**(f) Answer any one question :** **$2 \text{ X } 1 = 2$**

(i) If $P(A) = a$, $P(B)=b$, then show that $P(A/B)>a+b-1/b$

(ii) Find the sum of : $1+1/3!+1/5!+1/7!+$……………

**(g) Answer any one question :** **$2 \text{ X } 1 = 2$**

(i) If $x=1/z$, $y=f(x)$ and $d^2y/dx^2 = kz^3 dy/dz + z^4\ d^2y/dz^2$, then value of k is ---

(ii) If $\int f(x)\ dx = f(x)$, then $\int \{f(x)\}^2 dx$ is equal to ----------------

**(h) Answer any three question :** **$2 \text{ X } 3 = 6$**

(i) Evaluate : $\int dx/\sqrt{3-2x-x^2}$

(ii) The probability that a teacher will give a surprise test during any class Meeting is 1/5. If a student is absent on two days, what is the probability That he will miss atleast one test.

(iii) What is the condition for the expansion of Log (1+2x/1-2x) ? write down the expansion and from it, choosing a suitable value for x, find the expansion for log 3

(iv) If S & S' are the foci and P any point on this hyperbola, Prove that $CP^2 = SP.SP'$

(v) If $y = \sin x \log\{\tan(x/2)\}$, show that $(d^2y/dx^2)+y=\cot x$

## Group – C

**3. Answer the following questions as per instructions :**

**(a) Answer any two questions :** **$4 \text{ X } 2 = 8$**

(i) If each of the coefficient of the quadratic equation $ax^2+bx+c=0$ is selected Randomly from the integers 1,2 and 3. Find the probability that the Equation has equal roots.

(ii) If the letters of the word RAMESH be arranged at random, what is the probability That there are exactly three letters between A and E.

(iii) Among the examinees in an examination, 25%, 30% and 45% failed in statistics, In mathematics and in atleast one of statistics and mathematics respectively. An Examinee is selected at random. Find the probabilities that (a) he failed in Statistics only (ii) he passed in statistics if it is known that he failed in mathematics.

**(b) Answer any two questions :** **$4 \text{ X } 2 = 8$**

(i) The coordinates of foci of a hyperbola are $(0,\pm 4)$ and the length of its Latusrectum is 12 units; find its equation.

(ii) The coordinates of the vertices of a hyperbola are (9,2) and (1,2) and the Distance between the foci is 10 units. Find its equation and also the length Of latusrectum (iii)Prove that least focal chord of a parabola is the latus rectum.

**© Evaluate any two of the following integrals : 4 X 2 = 8**

(i) $\int 1/(x-a)^2(x-b)^3\,dx$ (ii) $\int x^2/x^4+x^2+1\,dx$ (iii) $\int x/(1-\cos x)^2\,dx$

**(d) Answer the following questions : 4 x 2 = 8**

(i) If A and B are any two events connected to a random experiment E, then Prove that $P(A\cap B)=P(A).P(B/A)$

(ii)If x is not equal to one, find the sum of $1+(1+x)/2!+(1+x+x^2)/3! + \text{------}\infty$

**(e) Answer any four questions : 4 x 4 = 16**

(i) Evaluate :$\int dx/(a\sin x + b\cos x)$

(ii) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ Show that $dy/dx = \sqrt{1-y^2}/\sqrt{1-x^2}$

(iii)The two lines $ty=x+t^2$ and $y+tx=2t+t^3$ intersect at the point P. Show that P Lies on the curve whose equation is $y^2=4x$

(iv)Find the greatest term in the expansion of $(2+3x)^5$ when x = ½

(v)A bag contains 10 balls out of which 5 are white, 2 are red and 3 are green. 2 balls are drawn at random. Find the probability that two drawn balls are Not white.

(vi)Identify the nature of the conic $4x^2+8x=5y+6$ and find out the length of its Latus rectum and equation of its directrix.

**Group – D**

**4. Answer the following questions as per instructions : 6 X 4 = 24**

**{Answer any one question from (a) and (b)}**

**{Answer any one from © and (d)}**

**{Answer any two from (e), (f), (g), (h)}**

a. (i) If the coefficient of (r+3)th term in the expansion of $(1+x)^{47}$ be the same as the coefficient of (3r+2)th term, find these two terms.

(ii)The coefficients of $5^{th}$, $6^{th}$ & $7^{th}$ terms in the expansion of $(1+x)^n$ are in AP find n. **3 + 3 = 6**

b. (i) For any two events A and B prove that

$P(A\cap B) \le P(A) \le P(AUB)\le P(A) + P(B)$

(ii) find the sum to infinity of the series $½(1/2)^2 + 2/3(1/2)^3 + \dots\dots\dots$

**3 + 3 = 6**

**C** (i) Find if possible the derivative of $\mathrm{Sec}^{-1}(1/8x^2-1)$ with respect to $\sqrt{1-x^2}$ at x=1/2

(ii)If $y=x^{n-1}\log x$, Prove that $x^2(d^2y/dx^2) +(3-2n)x\,dy/dx+(n-1)^2y=0$

**3 + 3 = 6**

(d) (i) If $f(x) = (1+x/2+x)^{3+2x} + \cos x$, find $f'(0)$

(ii) If $ax^2+2hxy+by^2=0$, prove that $( d^2y/dx^2) = (h^2-ab)/(hx+by)^3$

**3 + 3 = 6**

(e) (i) $\int dx/(1+x)\sqrt{1+x-x^2}$ (ii) $\int \log(x+1)/\sqrt{(x+1)}\,dx$ **3 + 3 = 6**

(f) (i) $\int(\log x-1)/\{1+(\log x)^2\}^2$ (ii) $\int dx/\mathrm{Sin}(x-a)\mathrm{Sin}(x-b)$ **3 + 3 = 6**

(g) (i) Integrate $1/\tan x + \cot x + \sec x + \operatorname{cosec} x$

(ii) $\int(3x+2)\sqrt{(x^2+x+1)}\,dx$ **3 + 3 = 6**

(h) (i) Find from first principle the derivative of log(cosx)

(ii) If f(x) has a finite derivate f'(a) ,at x=a show that

$$\lim_{X \to a} \frac{\{ Xf(a) - af(x) \}}{\{ x-a \}} = \{ f(a) - af'(a) \}$$

**3 + 3 = 6**