School of Math

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Time : 3hr. MM : 100

General Instructions:

1. *All questions are compulsory.*
2. *The question paper contains **29** questions.*
3. *Questions 1 – 4 in section A are very short – answer type questions carrying 1 marks each.*
4. *Questions 5-12 in Section B are short – answer type questions carrying 2 marks each.*
5. *Questions 13-23 in section C are long – answer – I type questions carrying 4 marks each.*
6. *Questions 24 – 29 in section D are long – answer – II type questions carrying 6 marks each.*

SECTION – A

Q1 Determine whether the relation R on N defined by , R = $\{(x, y) : y = x + 5, x < 4\}$, is reflexive, symmetric and transitive. 1

Q2 If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$, write the value of a – 2b. 1

Q3 If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 24$, then write the value of $|\vec{x}|$. 1

Q4 Let * be a binary operation on the set of all non- zero real numbers, given by a *b = $\frac{ab}{5}$ for all $a, b \in R - \{0\}$. Find the value of x, given that 2 * (x * 5) = 10 . 1

Section – B

Q5 Solve : cos $(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$. 2

Q6 If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, find α satisfying $0 < \alpha < \frac{\pi}{2}$ when A + A^T = $\sqrt{2}I_2$; Where A^T is transpose of A. 2

Q7 Prove that : $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 1, 0 < x < 1$ 2

Q8 If $y = x^4 - 10$ and if x changes from 2 to 1.99, what is the change in y? 2

Q9 Evaluate : $\int \frac{\sin(x-a)}{\sin(x+a)} dx$. 2

Q10 Show that the function $\tan^{-1} x + \tan^{-1} y = C$ is the general solution of the differential equation $(1+x^2)dy + (1+y^2)dx = 0$. 2

Q11 Find a vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k} and \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. 2

Q12 Two coins are tossed. What is the probability of coming up of two heads, if it is known that at least one head comes up? 2

Section – C

Q13 If a, b, c are pth , qth and rth terms respectively of a G.P. prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$. 4

Q14 Show that the function f defined by $f(x) = \begin{cases} 2x+3 & if \quad -3 \le x < -2 \\ x+1 & if \quad -2 \le x < 0 \\ -x+2 & if \quad 0 \le x \le 1 \end{cases}$ is not differentiable at x = -2 and x = 0 . 4

OR

Find k, if $f(x) = \begin{cases} k \sin \dfrac{\pi}{2}(x+1) & if \ x \le 0 \\ \dfrac{\tan x - \sin x}{x^3} & if \ x > 0 \end{cases}$ is continuous at x = 0.

Q15 If $y = e^{a\cos^{-1}x}, -1 \le x \le 1,$ show that $\left(1-x^2\right)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$. 4

Q16 Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes. 4

Q17 A magazine seller has 500 subscribes and collects annual subscription charges of Rs 300 per subscriber. She proposes to increase to increase and annual subscription charges and it is believed that for every increase of Rs 1, one subscriber will discontinue. What increase will bring maximum income to her? Make appropriate assumptions in order to apply derivatives to reach the solution. Write one important role of magazines in our lives. 4

Q18 Find $\int_0^{\pi/4} \dfrac{1}{\cos^3 x\sqrt{2\sin 2x}}\,dx$. 4

Q19 Solve the differential equation $\left(x^2 + 3xy + y^2\right)dx - x^2dy = 0$ given that y = 0 , when x = 1. 4

OR

Find the particular solution of the differential equation $\left(\tan^{-1} y - x\right)dy = \left(1+y^2\right)dx$, given that x = 1 when y = 0 .

Q20 Given that the vectors $\vec{a},\vec{b},\vec{c}$ from a triangle such that $\vec{a} = \vec{b} + \vec{c}$. Find p,q,r,s such that area of triangle is $5\sqrt{6}$, where $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}, \vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$. 4

Q21 Find the equation of plane passing through the points A(3,2,1), B (4,2,-2) and C (6,5,-1) and hence find the value of λ for which A (3,2,1), B(4,2,-2), C(6,5,-1) and D (λ ,5,5) are coplanar. 4

Q22 There are two bags A and B. Bag A contains 3 white and 4 red balls whereas bag B contains 4 white and 3 red balls. Three balls are drawn at random (without replacement) from one of the bags and are found to be two white and one red. Find the probability that these were drawn from bag B. 4

Section – D

Q23 In a group of 50 students in a camp, 30 are well – trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Find the mean of the distribution also. 6

Q24 Find the area bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7. 6

Q25 Let f : $N \to N$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$ is invertible (where S is range of f). Find the inverse of *f* and hence find $f^{-1}(31)$ and $f^{-1}(87)$. 6

OR

Show that the relation R defined by (a,b) R (c,d) $\Rightarrow$ a + d = b + c on $A \times A$, where A = {1,2,3,.....,10} is an equivalence relation. Hence write the equivalence class [(3,4)];a,b,c,d $\in A$.

Q26 Using elementary row operations, find the inverse of the following matrix: 6

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}.$$

OR

Two schools P and Q want to award their selected students on the values of Discipline, politeness and Punctuality. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to its 3,2, and 1 students with a total award money of Rs 1000. School Q wants to spend Rs 1500 to award its 4,1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is Rs 600,using matrices, find the award money for each value.

Q27 Evaluate : $\int_1^5 \left(|x-1| + |x-2| + |x-3|\right) dx$. 6

OR

Evaluate $\int_{-1}^{2} \left(e^{3x} + 7x - 5\right) dx$ as the limit of sum.

Q28 Find the coordinates of the points where the line through (3, -4, -5) and (2,3, -1) crosses the plane, passing through the points (2,2,1), (3,0,1) and (4, -1, 0). 6

Q29 The postmaster of a local postoffice whishes to hire extra helpers during the Deepawali season , because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives Rs 225 a day and a woman receives Rs 200 a day . How many men and women helpers should be hired to keep the pay roll at a minimum? Formulate an L.P.P.and solve it graphically.

Ans wer : 2 0 3 5 4 25 5 $\pm\frac{3}{4}$ 6 $\alpha=\frac{\pi}{4}$ 8 y = - 0.32 9

$x\cos 2a-\sin 2a\log|\sin(x+a)|+C$ 11 $\pm\left(\hat{i}-11\hat{j}-7\hat{k}\right)$ 12 1/3 14 or k = ½ 16 $\alpha=0,4$

17 Rs 100 18 $\frac{6}{5}$ 19 $-\frac{x}{y+x}=\log|x|-1$ or $xe^{\tan^{-1}y}=e^{\tan^{-1}y}\left(\tan^{-1}y-1\right)+2$

20 Either p = -8, q = 4, r = 2, s = -11 or p = 8 , q = 4 , r = 2 , s = 5 21 9x – 7y + 3z = 16, λ =4

22 3/5 23 6/5 24 6sq. units 25 $f^{-1}(x)=\frac{\sqrt{x-6}-3}{2}, f^{-1}(31)=1 and f^{-1}(87)=3$

26 $A=\begin{bmatrix}-7 & -9 & 10\\ -12 & -15 & 17\\ 1 & 1 & -1\end{bmatrix}$ or Rs 100, Rs 200, Rs 300 27 17 or $\frac{e^9-1}{3e^3}-\frac{9}{2}$

28 $\left(\frac{17}{9},\frac{34}{9},-\frac{5}{9}\right)$ 29 Minimum cost = Rs 2150, Number of men helpers = 6, Number of Women helpers = 4