

**School of Math**  
**SCF-33 SEC – 4 Market, Gurgaon. Ph:8586000650**  
**MATHEMATICS**  
**CLASS XII**

**Code : 13/E/17**

**Time: 3 hours**

**MM: 100**

**General Instructions:**

1. All questions are compulsory.
2. The question paper consists of **29** questions divided into four sections **A, B, C** and **D**. Section **A** comprises **4** questions of **one mark** each, Section **B** comprises **8** questions of **two marks** each, section **C** comprises of **11** question of **4** marks each and Section **D** comprises **6** questions of **six marks** each.
3. All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.
4. Use of calculator is not permitted. You may ask for logarithmic tables, if required.

**Section A**

- Q1 Let \* be a binary operation on N defined by  $a*b = a^b$  and let O be a binary operation on N defined by  $a \circ b = 3a + b$ , find  $(2*1) \circ 2$ . 1
- Q2 If  $f : R \rightarrow R$  be defined as  $f(x) = x^3 - 1$  and  $f^{-1}$  exists, find  $f^{-1}(-9)$ . 1
- Q3 For what value of  $k \in N$ ,  $\begin{vmatrix} k & 3 \\ 4 & k \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 0 & 1 \end{vmatrix}$ ? 1
- Q4 For what values of  $\lambda$ , are the vectors  $2\hat{i} + \lambda\hat{j} + \hat{k}$  and  $\hat{i} - 2\hat{j} + 3\hat{k}$  perpendicular to each other? 1

**SECTION – B**

- Q5 Evaluate :  $\tan^{-1} \left[ 2 \cos \left\{ 2 \sin^{-1} \left( \frac{1}{2} \right) \right\} \right]$ . 2
- Q6 If  $\vec{a}$  and  $\vec{b}$  are two vectors such that  $|\vec{a}| = 2$ ,  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 4$ , find  $|\vec{a} - \vec{b}|$ . 2
- Q7 If matrix  $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$  then show that  $A^2 = I$ . 2
- Q8 If  $y = \sqrt{\sin \sqrt{x}}$ , find  $\frac{dy}{dx}$ . 2
- Q9 Two cards are drawn without replacement from a pack of 52 cards. Find the probability that both cards are spades. 2
- Q10 Evaluate :  $\int \sin 4x \sin 2x dx$  2
- Q11 Solve the differential equation :  $\frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2x}$  2
- Q12 Show that the function  $f(x) = e^x$  strictly increases  $\forall x \in R$ . 2

**SECTION – C**

- Q13 Show that the function  $f(x) = \begin{cases} 6x^2 + x & \text{if } x \geq 2 \\ 25x & \text{if } x < 2 \end{cases}$  is differentiable at  $x = 2$ . 4

**OR**

Find the value of  $k$  so that the following function becomes continuous at  $x = 2$

$$\begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$$

- Q14 If  $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$  then using P.M.I prove that  $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in N$ . 4

- Q15 If  $y = 3 \sin \theta - 2 \sin^3 \theta$  and  $x = 3 \cos \theta - 2 \cos^3 \theta$ , find  $\frac{dy}{dx}$ . 4

- Q16 Prove that the function  $f(x) = \left[ \frac{4 \sin x}{2 + \cos x} - x \right]$  is an increasing function in  $\left(0, \frac{\pi}{2}\right)$ . 4

**OR**

Verify Rolle's theorem for the function  $f(x) = \log \left[ \frac{x^2 + ab}{x(a+b)} \right]$  on  $[a, b]$ .

- Q17 A rectangular sheet of tin 45cm broad is to be made into a box without top by cutting off equal square pieces from each corner and folding up the flaps. What should be the side of each square to be removed from each corner of the sheet so that the volume of the box is maximum. 4

- Q18 Evaluate :  $\int \left( \frac{x+2}{x+4} \right)^2 e^x dx$ . 4

- Q19 Solve the differential equation :  $x^2 \left( \frac{dy}{dx} \right) = x^2 + xy + y^2$  4

**OR**

Solve the differential equation  $\cos^2 x \left( \frac{dy}{dx} \right) + y = \tan x$

- Q20 If difference of two unit vectors is a unit vector, prove that magnitude of their sum is  $\sqrt{3}$ . 4

- Q21 Find image of a point A (5,8,1) in the line :  $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-8}{1}$ . 4

- Q22 In a class 5% of boys and 10% of girls have an I. Q. of more than 150. In the class 60% of the students are boys and rest are girls. If a student is selected at random and is found to be having I.Q. of more than 150, find the probability that the selected student is a boy. 4

- Q23 Calculate mean and variance and standard deviation of the following probability distribution. 6

x	0	1	2	3	4	5	6	7
p(x)	0	0.1	0.2	0.1	0.1	0.2	0.2	0.1

**Section – D**

- Q24 Given a non- empty set X, consider the binary operation  $\otimes : P(X) \times P(X) \rightarrow P(X)$  given by  $A \otimes B = A \cap B \forall A$  and  $B$  in  $P(X)$ ; Where  $P(x)$  is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in  $P(X)$  with respect to the operation  $\otimes$ . 6

OR

Prove that :  $\sin \left[ \frac{1}{2} \left\{ \sin^{-1} (2x\sqrt{1-x^2}) + \cos^{-1} (1-2y^2) \right\} \right] = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

Q25 Using properties of determinant show that :

6

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

OR

if  $A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 0 & 1 \\ 5 & 5 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \\ -3 & 4 & 1 \end{bmatrix}$  verify :  $(AB)^{-1} = B^{-1}A^{-1}$ .

Q26 using integration find the area of the region between the two curves

6

$x^2 + y^2 \leq 25$  and  $x + y \leq -5$ .

Q27

Using limit of sums method, evaluate  $\int_2^5 3^{(5x+4)} dx$

6

OR

Evaluate :  $\int \frac{dx}{\sin x + \sec x}$

Q28 Find the equation of a plane passing through the line of intersection of two planes  $3x + 2y + z + 2 = 0$  and  $x + y + z + 4 = 0$  and perpendicular to a plane  $5x - y - 3z + 7 = 0$ .

6

Q29 A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 57,600 to invest and has space for almost 20 items. A fan cost him Rs 3,600 and a sewing machine Rs 2,400. Profit on selling a fan and a sewing machine are Rs 220 and Rs 180 respectively. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Formulate this problem mathematically and solve it graphically.

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Ans 1 8 2 -2    3  $k=4 \in N$     4  $\lambda = \frac{5}{2}$     5  $\frac{\pi}{4}$     6  $|\vec{a}-\vec{b}| = \sqrt{5}$     8  $\frac{\cos \sqrt{x}}{4\sqrt{x}\sqrt{\sin \sqrt{x}}}$

9  $\frac{1}{17}$     10  $\frac{\sin 2x}{4} - \frac{\sin 6x}{12} + C$     11  $y = \tan x - x + c$     13 OR  $k=7$     15  $\frac{dy}{dx} = \cot \theta$

16 OR  $c = \sqrt{ab} \in (a,b)$     17 5cm    18  $\frac{xe^x}{x+4} + C$

19  $\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$  OR  $y = (\tan x - 1)x + C e^{-\tan x}$     21 Image (1,6,17)

$$AB = \begin{bmatrix} -5 & 16 & 6 \\ -1 & 10 & 3 \\ 6 & 32 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6} \begin{bmatrix} -5 & 13 & -1 \\ -1 & -1 & 1 \\ 10 & -20 & 2 \end{bmatrix}$$

22 3/7    23  $\mu = 4.1, \sigma^2 = 3.69, \sigma = 1.9$     25

$$B^{-1} = \frac{1}{19} \begin{bmatrix} 5 & 1 & -4 \\ 1 & 4 & 3 \\ 11 & -13 & -5 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{114} \begin{bmatrix} -66 & 144 & -12 \\ 21 & -51 & 9 \\ -92 & 256 & -34 \end{bmatrix}$$

26  $\left(\frac{25\pi}{4} - \frac{25}{2}\right)$  sq. units    27

$$\frac{1}{5 \log 3} [3^{29} - 3^{14}] \text{ OR } \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + (\sin x - \cos x)}{\sqrt{3} - (\sin x - \cos x)} \right| + \tan^{-1}(\sin x + \cos x) + C$$

28  $7x + 8y + 9z + 38 = 0$  and  $\lambda = -10$     29 Max. profit,  $Z_{\max} = \text{RS}3920$  deals with 8 fans and 12 sewing machines.