School of Math

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Time : 3hr. **MM : 100**

1. *All questions are compulsory.*
2. *The question paper contains **29** questions.*
3. *Questions 1 – 4 in section A are very short – answer type questions carrying 1 marks each.*
4. *Questions 5-12 in Section B are short – answer type questions carrying 2 marks each.*
5. *Questions 13-23 in section C are long – answer – I type questions carrying 4 marks each.*
6. *Questions 24 – 29 in section D are long – answer – II type questions carrying 6 marks each.*

Section – A

Q1 Let A = $\{1,2,3,4\}$. Let R be the equivalence relation on $A\times A$ defined by $(a,b)\,R(c,d)\,iff\,a+d=b+c$. Find the equivalence class (2,2). 1

Q2 If $A=[a_{ij}]$ is a matrix of order 3×3, such that |4A| = -256 and c_{ij} represents the cofactor of a_{ij}, then find $a_{12}.C_{12}+a_{22}.C_{22}+a_{32}.C_{32}$. 1

Q3 Find the direction cosines of the vector joining the points A (1,2, -3) and B(-1, -2, 1) , directed from A to B. 1

Q4 Determine whether the binary operation * on the set N of natural numbers defined by a*b = a^b is associative or not . 1

SECTION – B

Q5 If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of x. 2

Q6 Find the inverse of the matrix $\begin{bmatrix}8 & 3\\ 5 & 2\end{bmatrix}$. Hence find matrix A satisfying the matrix equation $A\begin{bmatrix}8 & 3\\ 5 & 2\end{bmatrix}=\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix}$. 2

Q7 Prove that if $\frac{1}{2}\le x\le 1$ then $\cos^{-1}x+\cos^{-1}\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right]=\frac{\pi}{3}$. 2

Q8 Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%. 2

Q9 Find $\int\frac{\left(x^2+1\right)e^x}{\left(x+1\right)^2}dx$ 2

Q10 Verify that given function (explicit or implicit) is a solution of the corresponding differential equation: y = x sin x : $xy'=y+x\sqrt{x^2-y^2}\ \left(x\ne 0\,and\ x>y\ or\ x<-y\right)$. 2

Q11 Find $\left|\vec{a}\right|\,and\left|\vec{b}\right|,if\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=8\,and\left|\vec{a}\right|=8\left|\vec{b}\right|$. 2

Q12 If A and B are two events such that P (A) = 0.4, P(B) = 0.8 and P (B/A) = 0.6, then find P (A/B). 2

Section – C

Q13 If $\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix} = 4$ then find the value of $\begin{vmatrix} a+2b & c+2d & p+2q \\ 2a+b & 2c+d & 2p+q \\ u & v & w \end{vmatrix}$. 4

Q14 Find the values of a and b, if the function *f* defined by $f(x) = \begin{cases} x^2+3x+a & if\ x \le 1 \\ bx+2 & if\ x > 1 \end{cases}$ is differentiable at x = 1. 4

OR

Determine the values of 'a' and 'b' such that the following function is continuous at x = 0.

$$f(x) = \begin{cases} \dfrac{x+\sin x}{\sin(a+1)x}, & if\ -\pi < x < 0 \\ 2 & if\ x = 0 \\ 2\dfrac{e^{\sin bx}-1}{bx}, & if\ x > 0 \end{cases}$$

Q15 If $y = \sqrt{x+1} - \sqrt{x-1}$, *prove* that $(x^2-1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - \dfrac{y}{4} = 0$. 4

Q16 Find points on the curve $\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$ at which the tangents are (i) parallel to x – axis (ii) parallel to y – axis . 4

OR

Find the intervals in which the function $f(x) = -3\log(1+x) + 4\log(2+x) - \dfrac{4}{2+x}$ is strictly increasing or strictly decreasing.

Q17 The total cost of producing x TV sets per day is Rs $(x^2+140x+100)$ and the price per set at which they may be sold is Rs (200 – 2x).Find the daily output to maximize the total profit. The workers employed for the production of TV sets are people from different communities. What values are highlighted here? 4

Q18 Integrate the function $\dfrac{5x+3}{\sqrt{x^2+4x+10}}$. 4

Q19 Find the particular solution of the differential equation 4

$\dfrac{dy}{dx} + y\cot x = 2x + x^2\cot x\,(x \ne 0)$ given that y = 0 when $x = \dfrac{\pi}{2}$.

OR

Show that the given differential equation is homogeneous and solve of it.

$$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}xdy.$$

Q20 If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ *and* $\hat{k}$ we have, $\vec{\alpha} = 3\hat{i} - \hat{j}, \vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ *and* $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. 4

Q21 Find the equation of the line which intersects the lines 4

$\dfrac{x+2}{1} = \dfrac{y-3}{2} = \dfrac{z+1}{4}$ *and* $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and passes through the point (1,1,1).

Q22 Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the large of the two numbers obtained. Find E(X). 4

Q23 If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup. 4

SECTION – D

Q24 Let * be a binary operation defined on $Q \times Q$ by (a,b) *(c,d) = (ac, b + ad) , where Q is the set of rational numbers. Determine, whether * is commutative and associative. Find the identity element for * and the invertible elements of $Q \times Q$.

OR

If the function f :R $\rightarrow R$ be defined by $f\left(x\right)=3x+4$ and g : R $\rightarrow R$ by g(x) = 5x – 4, then find fog and show that fog in invertible, Also show that $\left(fog\right)^{-1} = g^{-1}of^{-1}$.

Q25 If $A=\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations: 3x + 4y + 7z = 14, 2x – y + 3z = 4, x + 2y – 3z = 0 . 6

OR

If $A=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, find the inverse of A using elementary row transformations and hence solve the following matrix equation XA = [1 0 1].

Q26 Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola x^2 = 4y. 6

Q27 Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$ 6

OR

Evaluate $\int_{-2}^{2}\left(3x^2-2x+4\right)dx$ as the limit of a sum.

Q28 Find the distance of point $-2\hat{i}+3\hat{j}-4\hat{k}$ from the line $\vec{r}=\hat{i}+2\hat{j}-\hat{k}+\lambda\left(\hat{i}+3\hat{j}-9\hat{k}\right)$ measured parallel to the plane : x – y + 2z – 3 = 0 . 6

Q29 A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. 6

If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden? Formulate this problem as a L.P.P.an then solve it graphically.

kg per bag		
	Brand P	**Brand Q**
Nitrogen	3	3.5
phosphoric acid	1	2
potash	3	1.5
Chlorine	1.5	2

Answers : 1 {(1,1), (2,2), (3,3), (4,4)} 2 -4 3 $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$ 4 not associative 5 $\pm\frac{1}{\sqrt{2}}$ 6

$\begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ -1 & 2 \end{bmatrix}$ 8 0.03 x^3 m^3 9 $\int \frac{x^2+1}{(x+1)^2} e^x dx = \frac{x-1}{x+1} e^x + C$

11 $\frac{16\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{2}}{3\sqrt{7}}$ 12 3/10 13 -12

14 a = 3 and b = 5 OR we must have $\frac{2}{a+1} = 2 \Rightarrow a = 0;$ b may be any real number other than 0.

16 (0,5) and (0 , -5) ii) (2,0)and (-2,0) .OR decreasing in (-1,0) , increasing in [0, ∞).

17 10 TV sets.

18 $5\sqrt{x^2+4x+10} - 7\log\left|x+2+\sqrt{x^2+4x+10}\right| + C$ 19 $y = x^2 - \frac{\pi^2}{4\sin x}(\sin x \neq 0)$OR

$xy\cos\left|\frac{y}{x}\right| = C$ 20 $\overrightarrow{\beta_1} = \frac{1}{2}\left(3\hat{i} - \hat{j}\right) and \overrightarrow{\beta_2} = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$ 21 $\frac{x-1}{3} = \frac{y-1}{10} = \frac{z-1}{17}$ 22 $\frac{14}{3}$

23 0.95 24 15x – 8 25 x = 1, y = 1, z = 1 or

[0 1 0] 26 $\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

27 $\frac{\pi^2}{4}$ OR 32 28 $\frac{\sqrt{59}}{2}$ unit.

29 40 bags of brand P and 100 bags of brand Q; Minimum amount of nitrogen = 470 kg.