

AIMS INSTITUTE

UPSCA-125[®]

A PREMIER INSTITUTE OF MATHEMATICS FOR IIT, XI & XII SINCE 1998

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Subject – Mathematics**Class – XII****Time: 3 hours****By: Sandeep Shishodia****M.M. 100****General Instructions:**

- All questions are compulsory.
- The question paper consists of **26** questions divided into three sections A, B and C. Section A comprises of **6** questions of one mark each, Section B comprises of **13** questions of **four marks** each and Section C comprises of **7** questions of **six marks** each.
- All question in Section A are to be answered in one word, one sentence or as per exact requirement of the question.
- There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators and other electronic gadget is **not** permitted.

Section – A*(Questions number 1 to 6 carry 1 marks each)*

- If the vectors $\vec{a} = 2\hat{i} - \hat{j} + k\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = 3\hat{i} + \lambda\hat{j} + \hat{k}$ are coplanar, then find the value of λ .
- Find x, if $\begin{bmatrix} 5 & 3x \\ 2y & z \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 12 & 6 \end{bmatrix}^T$.
- If $\tan^{-1} \sqrt{3} + \cot^{-1} \theta = \frac{\pi}{2}$, then find θ .
- If a matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k .

5. If $f : \mathbf{R} \rightarrow \mathbf{R}$ be defined by $f(x) = (3 - x^3)^{1/3}$, then find $f \circ f(x)$.
6. What positive value of x makes following pair of determinants equal $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$?

Section-B

(Questions number 7 to 19 carry 4 marks each.)

7. $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors. Suppose $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between \vec{b} and \vec{c} is $\frac{\pi}{6}$,
prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

OR

Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{i} + \hat{k}$ and the other is perpendicular to \vec{b} .

8. Find the probability distribution of the number of doublets in four throws of a pair of dice. *Prateek respects his teachers very much and teachers do the same for him due to this he is getting extra advantage in his class, some of naughty students of his class have jealousy from Prateek they always try to pull his legs. You are the monitor of the class how you will handle this situation.*

9. Solve the differential equation $\left[\frac{e^{-2\sqrt{y}}}{\sqrt{y}} - \frac{x}{\sqrt{y}} \right] \frac{dy}{dx} = 1$; ($y \neq 0$) and $y(1) = 2$.

10. Solve the differential equation $\sqrt{1 + x^2 + y^2 + x^2 y^2} + xy \frac{dy}{dx} = 0$.

OR

Find the particular solution of the differential equation

$$\left[x \sin^2 \frac{y}{x} - y \right] dx + x dy = 0; y = \frac{\pi}{4} \text{ when } x = 1.$$

11. Evaluate $\int \frac{\sqrt{x^2 + a^2}}{x} dx$.

12. Evaluate $\int_{-1}^1 e^{\theta} d\theta$ as limit of sum.

13. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{4\sin^2 x + \cos^2 x} dx$.

14. Find the interval in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is

(i) increasing (ii) decreasing .

15. If $x = a\left(\cos\theta + \log \tan \frac{\theta}{2}\right)$ and $y = a \sin \theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

16. Find the value of k for which $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1} & \text{if } 0 \leq x < 1 \end{cases}$ is continuous at $x = 0$.

17. For the matrix $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find a and b such that $A^2 + aI = bA$.

OR

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying $AA' = 9I$, then find the value of a and b .

18. Let N be the set of all natural numbers and R be the relation on $N \times N$ defined by $(a,b)R(c,d)$ iff $ad(b+c) = bc(a+d)$. Prove that R is an equivalence relation.

19. Prove that $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right] = \sqrt{\frac{1+x^2}{2+x^2}}$.

OR

Solve the following equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

Section-C

(Question number 20 to 26 carry 6 marks each)

20. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$ find A^{-1} , and hence solve the following system of equations:

$$\begin{aligned} 2x + y + 3z &= 3 \\ 4x - y &= 3 \\ -7x + 2y + z &= 2. \end{aligned}$$

21. Find the vector and cartesian equation of the plane containing two lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \lambda(-2\hat{i} + 3\hat{j} + 8\hat{k})$.

22. Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

23. Prove that semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1} \sqrt{2}$.

24. Using integration find the area of the region included between the curves $y = x^2 + 1, y = x, x = 0$ and $y = 2$.

OR

Make the rough sketch of the region given below and find its area using integration : $\{(x, y) : 0 \leq y \leq x^2 + 3; y \geq 2x + 3; 0 \leq x \leq 3\}$.

25. Assume that the chance of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course

of meditation and yoga? *Why Meditation and Yoga is necessary and sufficient thing for peace in mind and for good health.*

OR

A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that all the balls are blue? *What good values we should learn from games and sports?*

26. An aero plane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit? *How one should respect the hard earn money of parents/guardians in a best economical way.*

AIMS

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