Most Important Questions

1. Show that the relation R in the set Z of integers given by $R=\{(a,b): 2\ divides\ a-b\}$ is an equivalence relation. Also find its all possible equivalence classes.

2. Check whether the relation R in R of real numbers defined by $R=\{(a,b): a \le b^3\}$ is reflexive, symmetric or transitive.

3. Show that the relation R defined by $(a,b)R(c,d) \Leftrightarrow a+d=b+c$ on the set $N \times N$ is an equivalence relation.

4. $f: R \to R,\ g: R \to R$ given by $f(x)=[x],\ g(x)=|x|$, then find $(fog)\left(\frac{-2}{3}\right)$ and $(gof)\left(\frac{-2}{3}\right)$.

5. Show that the function $f: N \to N$ defined by $f(x)=\begin{cases} \frac{n+1}{2} & ,n\ is\ odd \\ \frac{n}{2} & ,n\ is\ even \end{cases}$ is not bijective.

6. Consider $f: R_+ \to [-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible with $f^{-1}(y)=\frac{\sqrt{y+6}-1}{3}$.

7. Discuss the commutativity and associativity of the binary operation * defined on Q by the rule $a*b=a-b+ab$ for all $a,b,c \in Q$.

8. Let R^+ be the set of all positive reals. Define an operation 'o' on R^+ by $a \text{ o } b=\frac{ab}{4}, \forall\ a, b \in R^+$. Show that the operation 'o' is commutative as well as associative. Also find the identity element and the inverse of *a*.

9. Find the value of $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$.

10. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{2}\right)$

11. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\le x\le 1$

12. Prove that $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$.

13. Prove that $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$.

14. Solve : $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$.

15. If *A* and *B* are symmetric matrices, determine whether *AB* – *BA* is symmetric or skew-symmetric matrix.

16. Express $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrices.

17. Obtain the inverse of the matrix $A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operations.

18. There are three families A, B and C. The number of men, women and children in these families are as under :

	Men	Women	Children

Family A	2	3	1
Family B	2	1	3
Family C	4	2	6

Daily expenses of men, women and children are Rs.200, Rs.150 and Rs.200 respectively. Only men and women earn and children do not. Using matrix multiplication, calculate the daily expenses of each family. What impact does more children in the family create on the society ?

19. If A is a square matrix of order 3×3 such that $|A| = 5$, then find $|4A|$.

20. If A is a square matrix of order 3×3 such that $|A| = 5$, then find $|adjA|$.

21. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$.

22. Show that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$.

23. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$.

24. Prove that $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)^3$.

25. If $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$, then prove that $a+b+c=0$ or $a=b=c$.

26. Solve the system of equations by using matrix method :

$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

27. Solve the following system of equations by matrix method where $x, y, z \neq 0$.

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$$
$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$$
$$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

28. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$$x - y + 2z = 1$$
$$2y - 3z = 1$$
$$3x - 2y + 4z = 2$$

29. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school wants to award Rs.x each, Rs.z each for the three

respective values to 3, 2 and 1 students respectively with a total award money of Rs.1600. School B wants to spend Rs.2300 to award its 4, 1 and 3 students on the respective values(by giving the same award money to the three values as before). If the total amount of award for one prize on each value is Rs.900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

30. Show that the function *f* defined by $f(x)=\begin{cases}\frac{|x-4|}{x-4}, x\neq 4\\ 0, x=4\end{cases}$ is continuous every where except at $x = 4$.

31. Determine the value of *k* for which the function $f(x)=\begin{cases}\frac{1-\cos 2x}{2x^2}, x\neq 0\\ k, x=0\end{cases}$ is continuous at $x = 0$.

32. Determine the values of a, b and c for which the function $f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, x<0\\ c, x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}}, x>0\end{cases}$ is continuous at $x = 0$.

33. Prove that the function *f* given by $f(x)=|x-1|, x\in R$ is continuous at $x = 1$ but not differentiable at $x=1$.

34. Find $\frac{dy}{dx}$, if $y=(\sin x)^{\cot x}+(\sin x)^{\sec x}$.

35. Find $\frac{dy}{dx}$, if $x^y+y^x=a^b$.

36. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

37. If $y=\sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)$ and $0<x<1$, then find $\frac{dy}{dx}$.

38. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for -1 < x < 1, prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.

39. If $\cos y=x\cos(a+y)$, with $\cos a\neq\pm 1$, prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$.

40. If $x^y=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$.

41. Find the derivative of $\log_e(\sin x)$ w.r.t $\log_e(\cos x)$.

42. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\frac{d^2y}{dx^2}$.

43. If $y=(\tan^{-1}x)^2$, show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$.

44. If $y=\sqrt{x+1}-\sqrt{x-1}$, prove that $(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-\frac{1}{4}y=0$

45. Verify Rolle's Theorem for the function $y=x^2+2$ in the interval [-2, 2].

46. Use Lagrange's Mean Value Theorem to determine a point on the curve $y=\sqrt{x-2}$ at the tangent is parallel to the chord joining the points (2,0) and (3,1).

47. A man 2metres high walks at a uniform speed of 5km/hr away from a lamp post 6 metre high. Find the rate at which the length of his shadow increases.

48. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 4 m away from the wall.

49. Sand is pouring from a pipe at the rate of 12 cc/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand-cone increasing when the height is 4 cm.

50. A particle moves along the curve $y=\frac{2}{3}x^3+1$. Find the point on the curve at which *y*-coordinate is changing twice as fast as *x*-coordinate.

51. Prove that the function $f(x)=\frac{x^3}{3}-x^2+9x,\ x\in[1,2]$ is strictly increasing. Hence find the minimum value of $f(x)$.

52. Show that the line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-\frac{x}{a}}$ at the point where the curve crosses the y-axis.

53. For the function $f(x)=-2x^3-9x^2-12x+1$, find the intervals in which *f(x)* is i) increasing ii) decreasing.

54. Find the intervals in which $f(x)=\sin x-\cos x$, where $0 < x < 2\pi$ is increasing or decreasing.

55. Find the equation of the tangent and normal to the curve $y = \sin^2 x$ at the point $\left(\frac{\pi}{4},\frac{3}{4}\right)$.

56. Find the point on the curve $y=3x^2-12x+6$ at which the tangent is parallel to the *x*- axis. Also find the equation of tangent at that point.

57. Find the points on the curve $y=x^3-2x^2-x$ at which the tangent lines are parallel to the line $y=3x-2$.

58. Prove that the curves $x=y^2$ and $xy=k$ cut at right angles if $8k^2=1$.

59. Use differentials to approximate $\sqrt{50}$.

60. Find the approximate change in the volume of a sphere of radius 10cm if there is an error of 0.1 cm in measuring its radius.

61. Find the absolute maximum and minimum values of the function defined by $f(x)=\frac{1}{3}x^3-3x^2+5x+8$ in [0,4].

62. Show that a closed right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of its base.

63. A figure consists of semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.

64. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and circle is minimum.

65. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off, so that the volume of the box is the maximum possible? Also find the maximum volume.

66. Prove that the semi vertical angle of a cone of maximum volume and of given slant height l is $\tan^{-1}\sqrt{2}$.

67. Show that a conical tent of given capacity will require least amount of canvas if its height is $\sqrt{2}$ times the radius of its base.

68. Show that the volume of the greatest cylinder which can be inscribed in a right circular cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

69. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

70. A jet plane of enemy I s flying along the curve $y=x^2+2$ and a soldier is placed at the point (3,2). Find the minimum distance between the soldier and the jet plane.

71. Show that the right triangle of maximum area that can be inscribed in a circle is an isosceles triangle.

72. A manufacturer can sell x items at a price of Rs.$\left(5-\frac{x}{100}\right)$ each. The cost price is Rs. $\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to earn maximum profit.

73. Evaluate: $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$.

74. Evaluate: $\int e^x \frac{x^2+1}{(x+1)^2}.dx$.

75. Evaluate: $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$.

76. Evaluate: $\int \frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta$.

77. Evaluate: $\int \frac{x+2}{2x^2+6x+5}dx$.

78. Evaluate: $\int \frac{x^4}{(x-1)(x^2+1)}.dx$.

79. Evaluate: $\int \frac{2x}{(x^2+1)(x^2+4)}.dx$.

80. Evaluate: $\int \frac{\sin x}{(4+\cos^2 x)(2-\sin^2 x)}.dx$.

81. Evaluate: $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}.dx$.

82. Evaluate: $\int \frac{1}{x(x^4+16)}.dx$.

83. Evaluate: $\int \frac{e^{\tan^{-1}x}}{(1+x^2)^2}.dx$.

84. Evaluate: $\int \frac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^4}.dx$.

85. Evaluate: $\int \frac{x^2+1}{x^4+1}.dx$.

86. Evaluate: $\int \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$.

87. Evaluate: $\int \frac{1-\cos x}{\cos x\,(1+\cos x)}.dx$.

88. Evaluate: $\int \frac{1}{\sin^4 x+\sin^2 x.\cos^2 x+\cos^4 x}.dx$.

89. Find $\int_0^2 (x^2+1).dx$ as the limit of a sum.

90. Evaluate $\int_0^2 e^x dx$ as the limit of a sum.

91. Evaluate : $\int_{-1}^{2} \left|x^3-x\right|.dx$.

92. $\int_{-1}^{3/2} \left|x\sin \pi x\right|.dx$

93. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx$.

94. Evaluate : $\int_0^{\frac{\pi}{2}} \log(\sin x).dx$.

95. Evaluate : $\int_0^{\frac{\pi}{4}} \log(1+\tan x).dx$.

96. Evaluate : $\int_0^{\pi} \frac{xdx}{a^2\cos^2 x+b^2\sin^2 x}$.

97. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos x}.dx$.

98. Evaluate: $\int \frac{x^2}{\left(x\sin x+\cos x\right)^2}.dx$.

99. Evaluate: $\int_0^{\pi} \frac{x}{1+\cos\alpha.\sin x}dx$.

100. Evaluate: $\int_0^{\pi/2} \frac{x\sin x.\cos x}{\sin^4 x+\cos^4 x}dx$.

101. Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x.\operatorname{cosec} x}dx$.

102. Find the area bounded by the curve $x^2=4y$ and the line $x=4y-2$.

103. Find the area of the region bounded by the two parabolas $y=x^2$ and $x=y^2$.

104. Find the area lying above x-axis and included between the circle $x^2+y^2=8x$ and the parabola $y^2=4x$.

105. Using integration, find the area of the region bounded by the triangle whose vertices are (1,0), (2,2) and (3,1).

106. Using integration, find the area of the triangular region whose sides have equations $y = 4x + 5, x + y = 5$ and $x - 4y + 5 = 0$.

107. Find the area bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

108. Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$.

109. Find the area of the region bounded by the curve $y = \sqrt{1 - x^2}$, line $y = x$ and the positive x-axis.

110. Find the area of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$.

111. Using integration, find the area bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$.

112. Sketch the graph of $y = |x + 1|$. Evaluate $\int_{-3}^{1} |x + 1| dx$. What does this value represent on the graph.

113. Determine the order and degree of each of the following differential equations:

i) $x\frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = y^2$ ii) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = 5\frac{d^2y}{dx^2}$

114. Show that $y = ae^{2x} + be^{-x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

115. Form the differential equation of the family of the curve $y = a\cos(x + b)$ where a, b are arbitrary constants

116. Find the differential equation of all circles which pass through the origin and whose centre lies on y-axis.

117. The surface area of a balloon being inflated changes at a constant rate. If initially, its radius is 3 units and after 2 seconds it is 5 units, find the radius after t seconds.

118. Find the equation of the curve passing through the point (1,0) if the slope of the tangent to the curve at the point (x, y) is $\frac{y-1}{x^2 + x}$.

119. Solve : $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$.

120. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that $y = 1$ when $x = 0$.

121. Solve : $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$.

122. Solve: $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to the initial condition $y(0) = 0$.

123. Solve : $y\,dx + (x - y^3)dy = 0$.

124. Solve : $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$.

125. Find a unit vector in the direction of the resultant of the vectors $\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} + \hat{j} - 2\hat{k}$ and $-\hat{i} - \hat{j} - 3\hat{k}$.

126. Find the value of λ so that th e vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

127. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

128. Show that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

129. Find the projection of the vector $2\hat{i} - \hat{j} - \hat{k}$ in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$.

130. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular $\vec{\alpha}$.

131. Find a vector of magnitude 9 which is perpendicular to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$.

132. Given $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, find $|\vec{a} \times \vec{b}|$.

133. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = 3\hat{j} - \hat{k}, \vec{c} = 7\hat{i} - \hat{k}$, find a vector $\vec{d}$ which is perpendicular both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 1$.

134. Find the area of the triangle whose vertices are given by (1, 1, 0), (3, -1, 1) and (2, 0, -1).

135. Find the area of the parallelogram whose sides are represented by the vectors $\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$.

136. Find the equation of the line in vector and Cartesian form which is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$ and passing through the point (1, 2, 3).

137. Find the equation of the perpendicular from the point (3, -1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the foot of the perpendicular and the length of the perpendicular.

138. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

139. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Find their point of intersection.

140. Find the distance between the lines $\frac{x-2}{-2} = \frac{y-3}{6} = \frac{z-4}{-3}$ and $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

141. Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{2}$.

142. Find the equation of the plane which bisects the line joining the points (2,1,5) and (0,3,1) at right angles.

143. Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to each of the plane $x + 2y + 3z - 7 = 0$ and $2x - 3y + 4z = 0$.

144. Find the image of the point (1, 3, 4) in the plane $x - y + z = 5$.

145. Find the vector equation of the line passing through the point (1,2,3) and parallel to the planes $\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$.

146. Find the equation of the line passing through the point (1, 3, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$.

147. Find the distance of the point (1, -2 , 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x+1}{2}=\frac{y+3}{3}=\frac{z+1}{-6}$.

148. Find the distance of the point (1, -2 , 3) from the line $\frac{x+1}{2}=\frac{y+3}{3}=\frac{z+1}{-6}$ measured parallel to the plane $x-y+z=5$.

149. Find the equation of the plane containing the coplanar lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z=7}{2}$.

150. Find the equation of the plane passing through the point (1, 1, 1) and the intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$.

151. Find the vector equation of the plane passing through the points (1, 1, 2) and (-1, 2, 3) and perpendicular to the plane $x-2y+3z-4=0$.

152. Find the value of x such that the points A(3,2,1), B(4,x,5), C(4,2,-2) and D(6,5,-1) are coplanar.

153. Find the distance between the two planes $2x+3y+4z=4$ and $4x+6y+8z=12$.

154. A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of the triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

155. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs.5760.00 to invest and has space for at most 20 items. A fan costs him Rs.360.00 and a sewing machine Rs.240.00. His expectation is that he can sell a fan at a profit of Rs.22.00 and a sewing machine at a profit of Rs.18.00. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit ? Translate this problem mathematically and then solve it.

156. A firm manufactures two types of products A and B and sells them at a profit of Rs.5 per unit of type A and Rs.3 per unit of type B. Each product is processed on two machines M and N. One unit of type A requires one minute of processing time on M and two minutes of processing time on N where as one unit of type B requires one minute of processing time on M and one minute on N. Machines M and N are respectively available for atmost 5 hours and 6 hours a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

157. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs.50 per kg to purchase Food 'I' and Rs.70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimize the cost of such a mixture and hence solve it.

158. Two tailors A and B earn Rs.300 and Rs.400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

159. There is factory located each of the two places P and Q. From these locations, a certain commodity is delivered to each of the depots situated at A, B and C. the weekly requirements of the depots are respectively 5,5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below :

	Cost in Rs.

	A	B	C
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum?

160. A die is rolled twice and the sum of the numbers on them is observed to be 7. What is the conditional probability that the number 2 has appeared at least once?
161. A die whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even". And B be the event "number obtained is red". Find if A and B are independent events.
162. A person has under taken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike and 0.32 that the construction job will be completed on time if there is strike. Determine the probability that the construction job will be completed on time.
163. Two students can solve a problem with the probabilities 2/5 and 1/3. If a problem is given to them, find the probability that i) the problem is solved and ii) only one of them solves the problem.
164. In answering a question on a multiple choice test a student either knows the answer or guesses. Let ¾ be the probability that he knows the answer and ¼ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability ¼. What is the probability that a student knows the answer given that he answered it correctly?
165. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. Do you also agree that the value of truthfulness leads to more respect in the society?
166. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all the students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one students is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.
167. A random variable X has the fallowing probability distribution values of X :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find each of the fallowing : (i) k (ii) $P(X < 6)$ (iii) $P(X \geq 6)$ (iv) $P(0 < X < 5)$

168. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation and variance of X.
169. In a hurdle race, a player has to cross 10 hurdles. The probability that he will cross each hurdle is 5/6. What is the probability that he will knock down fewer than two hurdles?
170. In a hockey match, both teams A

A and B scored same number of goals up to the end of the game, so decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared winner. If the captain of A was asked to start, find their respective probabilities of winning match and state whether the decision of the referee was fair or not?