

**Guess Paper – 2011**  
**Class – XII**  
**Subject – Mathematics**

Time: 3 hours

M. Marks:100

**General Instructions:**

1. All questions are compulsory.
  2. The questions paper consists of 29 questions divided into three sections A, B and C. Section A consists of 10 questions of one mark, section B consists of 12 questions of four marks each and section C consists of seven questions of six marks each.
  3. All questions in section A are to be answered in word, one sentence or as per the exact requirement of the question.
  4. There is no overall choice. However, internal choice has to be provided in four questions of four marks each and two questions of six marks each. You have to attempt only one of the alternatives in all such questions.
  5. Use of calculators is not permitted.
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Q1. Consider  $f: \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = 4x + 3$ . Show that  $f$  is one – one

Q2. Find the value of  $x$  and  $y$  if  $3 \begin{pmatrix} 2 & 1 \\ 3 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 4 \end{pmatrix}$

Q3. Find the principal value of  $\operatorname{cosec}^{-1}(-\sqrt{2})$

Q4. If  $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$ , determine whether  $A - A'$  is symmetric or skew symmetric

Q5. If  $A$  is a square matrix of order 3, such that  $|\operatorname{adj}A| = 81$ , find  $|A|$

Q6. Find the intervals in which the function  $f$  given by  $f(x) = \sin x + \cos x$ ,  $x \in [0, 2\pi]$  is strictly decreasing

Q7. Find the value of  $\int \frac{dx}{\sin^2 x \cos^2 x}$

Q8. If  $\vec{a} = i + 2j - k$  and  $\vec{b} = 3i + j - 5k$ , find a unit vector in the direction of  $\vec{a} + \vec{b}$

Q9. Find the area of the parallelogram whose adjacent sides are given by  $\vec{a} = i - j + 3k$  and  $\vec{b} = 2i - 7j + k$

Q10. Find the intercepts cut off by the plane  $2x + y - z = 5$

Section: B

Q11. Let  $A = N \times N$  and  $*$  be the binary operation on A defined by  $(a, b) * (c, d) = (a+c, b+d)$ . Show that  $*$  is commutative and associative. Find the identity element for  $*$  on A.

Q12. a) Write the following function in simplest form

$$\tan^{-1} \left( \frac{\sqrt{1 - \cos x}}{\sqrt{1 + \cos x}} \right), \quad x < \pi$$

b) Prove that:

$$2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$$

OR

Solve for x:

$$\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$$

Q13. By using properties of determinants, show that  $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

OR

If  $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = 0$ , Show that  $1+pxyz=0$  where

$$x \neq y \neq z$$

Q14. Discuss the continuity of the following function at  $x=0$

$$f(x) = \begin{cases} \frac{x^4 + 2x^3 + x^2}{\tan^{-1} x} & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

Q15. If  $x=a(t+\sin t)$ ,  $y=a(1-\cos t)$  Find  $\frac{dy}{dx}$  at  $t=\frac{\pi}{2}$

OR

If  $y=x \cos(a+y)$ , prove that  $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

Q16.If the gradient of the curve  $2y^2=ax^2+b$  at  $(-1,1)$  is  $-1$ , find  $a$  and  $b$ .

Q17.Evaluate:  $\int \cos(x+a)\cos(x+b) dx$

$$\cos(x+a)\cos(x+b)$$

Q18. Integrate the following:

$$\int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

Q19. Integrate the function:

$$\frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$$

OR

Evaluate the following integral as a limit of sum:

$$\int_1^4 [(x^2] - x) dx$$

Q20. Show that the points  $A(1,-2,-8)$ ,  $B(5,0,-2)$  and  $C(11,3,7)$  are collinear, and find the ratio in which  $B$  divides  $AC$ .

Q21 Find the values of p so that lines  $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$  and  $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$  are at right angles.

Q22. Probability of solving a specific problem independently by A and B are  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively. If both try to solve the problem independently, find the probability that

- i) Problem is solved
- ii) Exactly one of them solves the problem

Section: C

Q23. Using matrices, solve the following system of equations:

$$2x-3y+5z=11$$

$$3x+2y-4z=-5$$

$$x+y-2z=-3$$

Q24. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is  $\frac{8}{27}$  of the volume of the sphere. OR

A square piece of tin of side 18cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is maximum?

Q25. Find the area of the smaller region bounded by  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line  $\frac{x}{3} + \frac{y}{2} = 1$

Q26. Find the general solution of  $\frac{dy}{dx} + 2y = \sin x$

Q27. There are two types of fertilizers  $F_1$  and  $F_2$ .  $F_1$  consists of 10% nitrogen and 6% phosphoric acid and  $F_2$  consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for the crop. If  $F_1$  costs Rs6/kg and  $F_2$

costs Rs5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost ?

Q28. There are three coins. One is two headed coin, another is biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows head, what is the probability that it was the two headed coin?

Q29. Find the equation of the plane through intersection of the plane  $x+2y-z=4$  and the line

$$\frac{x-1}{2} = \frac{y+3}{4} = \frac{z}{1} \text{ and parallel to the plane } \vec{r} \cdot (3\hat{i} - k\hat{j}) = 2$$

OR

Find the equation of the plane through the intersection of the planes  $2x + y - 3z = 4$  and  $3x + 4y + 8z - 1 = 0$  and making equal intercepts on the coordinates axes.

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