School Of Math

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MATHEMATICS

CLASS XII **Code : 12E /17**

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **29** questions divided into three sections **A, B,C** and **D**. Section **A** comprises **4** questions of **one mark** each, Section **B** comprises **8** questions of **two marks** each , Section **C** comprises 11 questions of **four marks** each and Section **D** comprises 6 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

Section – A

Q1 If cos $\left(\tan^{-1} x + \cot^{-1}\sqrt{3}\right) = 0$, then find the value of x. 1

Q2 If matrix A = $\left[a_{ij}\right]_{2\times 2}$, where a_{ij} = 1, if $i \neq j$ *and* $= 0$ if i = j , then find A^2 . 1

Q3 Find the sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane 2x – 2y + z = 5 . 1

Q4 If $f : N \to R$ be the function defined by $f(x) = \frac{2x-1}{2}$ *and* $g : Q \to R$ be another function defined by g(x) = x + 2. then Find, (gof) $\left(\frac{3}{2}\right)$. 1

SECTION – B

Q5 Let A = R – {3} , B = R – {1}. If *f* : A $\to B$ be defined by f(x) = $\frac{x-2}{x-3}$ $\forall x \in A$. Then show that f is bijective. 2

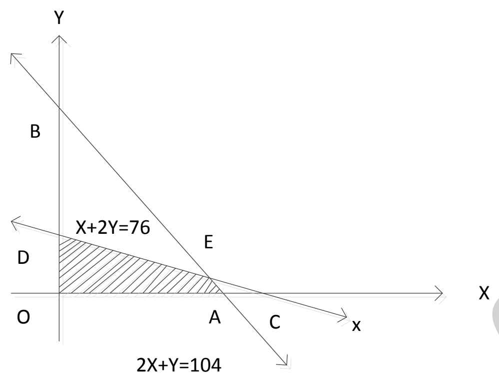
Q6 If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$ and $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$ then find A – B . 2

Q7 If $\sin\sqrt{x} + \cos^2\sqrt{x} = y$, *find* $\frac{dy}{dx}$. 2

Q8 Find the solution of $\frac{dy}{dx} = 2^{y-x}$. 2

Q9 Find the position vector of a point A in space such that $\overrightarrow{OA}$ is inclined at $60^0 to OX$ 45^0 to OY and $\left|\overrightarrow{OA}\right| = 10$ units . 2

Q10 Determine the maximum value of Z = 3x + 4y, if the feasible region (shaded) for a LPP is shown in following figure. 2



Q11 The probability that at least one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P\left(\overline{A}\right) + P\left(\overline{B}\right)$. 2

Q12 Find the approximate value of $(1.999)^5$. 2

Section – C

Q13 Find the number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$. 4

OR

If a + b + c ≠ 0 and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0,$ then prove that a = b = c.

Q14 Using mean value theorem , prove that there is a point on the curve y = $2x^2 - 5x + 3$ between the points A(1,0) and B (2,1), where tangent is parallel to the chord AB. Also, find that point. 4

Q15 Find the equation of the normal lines to the curve $3x^2 - y^2 = 8$ which are parallel to the line x + 3y =4. 4

OR

If x and y are the sides of two squares such that $y = x - x^2$, then find the rate of change of the area of second square with respect to the area of first square.

Q16 Evaluate $\int_1^4 \left(x^2 + e^{4x}\right)dx$ as limit of sums. 4

Q17 Find the area of the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$. 4

Q18 If y (t) is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and y(0)=-1 ,then show that y(1) = -1/2 . 4

OR

Find the equation of a curve passing through origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$.

Q19 Find the altitude of the parallelopiped determined by the vectors, $\vec{a}, \vec{b}, \vec{c}$ if the base is taken to be the parallelogram determined by $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - \hat{k} and \vec{c} = \hat{i} + \hat{j} + 3\hat{k}$ 4

Q20 Prove that the line through A(0,-1,-1) and B (4,5,1) intersect the line through C(3,9,4) and D (-4,4,4). 4

Q21 A firm has to transport 1200 packages using large vans which can carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is Rs 400 and each small van is Rs 200. Not more than Rs 3000 is to be spent on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost. 4

Q22 Let X be a discrete random variable whose probability distribution is defined as follows. 4

$$P(X = x) = \begin{cases} k(x+1) & for \quad x = 1,2,3,4 \\ 2kx, & for \quad x = 5,6,7 \\ 0 & otherwise \end{cases}$$

where , k is a constant . Calculate

i) the value of k. ii) E(X) iii) standard deviation of X.

Q23 A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in box of 10 each. From a single box, find the probability that 4

i) none of the bulbs is defective ii) exactly two bulbs are defective.

iii) more than 8 bulbs work properly.

SECTION – D

Q24 Evaluate : $\int_0^{\pi/2} \frac{dx}{\left(a^2\cos^2 x + b^2\sin^2 x\right)^2}$ 6

OR

Evaluate $\int_0^{1/2} \frac{dx}{\left(1+x^2\right)\sqrt{1-x^2}}$

Q25 Let Q be the set of all rational numbers. Define an operation *on Q- {-1} by a*b = a + b + ab. Show that 6
i) '*' is a binary operation on Q – {-1} .ii) '*' is commutative iii) '*'is associative iv) Zero is the identity element in Q – {-1}for * v) $a^{-1} = \left(\frac{-a}{1+a}\right), where\ a \in Q - \{-1\}$

OR

If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta, prove that \frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$.

Q26 Find x,y and z, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A'= A^{-1}. 6

Q27 Find the value of $\frac{dy}{dx}, if y = x^{\tan x} + \sqrt{\frac{x^2+1}{2}}$. 6

Q28 Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible when revolved about one of its sides. Also, find the maximum volumes. 6

Q29 $\overrightarrow{AB} = 3\hat{i} - \hat{j} + \hat{k} and \overrightarrow{CD} = -3\hat{i} + 2j + 4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{i} + 7\hat{j} + 4\hat{k} and -9\hat{i} + 2\hat{k}$, respectively . Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both. 6

Ans : 1 . $x = \sqrt{3}$ 2. I 3. $\frac{\sqrt{2}}{10}$ 4 none of these 6 ½ I 7 $\frac{1}{2\sqrt{x}}\left[\cos\left(\sqrt{x}\right) - \sin\left(2\sqrt{x}\right)\right]$ 8 $2^{-x} - 2^{-y} = K$ 9 $5\hat{i} + 5\sqrt{2}\hat{j} + 5\hat{k}$ 10 196 11 1.1 12 31.920 13 1

14 Hence $\left(\frac{3}{2}, 0\right)$ is the point on the curve y = $2x^2$ – 5x + 3 between the points A (1,0) and B (2,1) , where tangent is parallel to the chord AB. 15 ± 8 or $2x^2$ – 3x + 1 16 $21 + \frac{1}{4}e^4\left(e^{12} - 1\right)$ 17 $\frac{9}{2} sq.units$ 18 or $y = \frac{4x^3}{3\left(1+x^2\right)}$ 19 Altitude = $4/\sqrt{38}$ 21 $x \geq 0, y \geq 0$ 22 1.7 23 $\frac{59(49)^9}{(50)^{10}}$

24 $\frac{\pi}{4}\left(\frac{a^2+b^2}{a^3b^3}\right)$ or $\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$ 26 $x=\pm\frac{1}{\sqrt{2}}, y=\pm\frac{1}{\sqrt{6}} z=\pm\frac{1}{\sqrt{3}}$

27 $x^{\tan x}\left[\frac{\tan x}{x}+\log x.\sec^2 x\right]+\frac{x}{\sqrt{2\left(x^2+1\right)}}$ 28 $864\pi cm^3$

29 $-6\hat{i}-15\hat{j}+3\hat{k}$