# CLASS XII
# SAMPLE PAPER
# MATHS

## DIFFERENTIABILITY-APPLICATIONS

*******-EASY AND MODERATE
*******-DIFFICULT

**1MARK**

1. If a line y = x + 1 is a tangent to the curve $y^2 = 4x$ ,find the point the of contact ?
2. Find the point on the curve $y = 2x^2 – 6x – 4$ at which the tangent is parallel to the x – axis
3. Find the slope of tangent for y = tan x + sec x at x = π/4
4. Show that the function f(x) == $x^3 – 6x^2 + 12x - 99$ is increasing for all x.
5. Find the maximum and minimum values, if any of $f(x) = |\cos 2x| + 3$
6. For the curve $y = 3x^2 + 4x$, find the slope of the tangent to the curve at the point x = -2.
7. Find a point on the curve $y = x^2 – 4x - 32$ at which tangent is parallel to x-axis.
8. Find a, for which f(x) = a(x+sinx)+a is increasing .
9. The side of a square is increasing at 4 cm/minute. At what rate is the area increasing when the side is 8 cm long?
10. Find the point on the curve $y = x^2 - 7x + 12$, where the tangent is parallel to *x*-axis.

**4 MARKS**

1. *Find the intervals in which the function* $f(x) = 2\log(x-2) - x^2 + 4x + 1$ *is increasing or decreasing* *******
2. Find the intervals in which the function $f(x) = x^3 - 6x^2 + 9x + 15$ is ( i ) increasing ( ii ) decreasing. *******
3. Find the equation of the tangent line to the curve x = θ + sinθ, y = 1+cosθ a=π/4*******

4. Prove that $y = \dfrac{4\sin\theta}{2+\cos\theta}$ is increasing in [o, π/2]
5. Prove that curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles If $c^4 = 32\, a^4$*******

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6. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}\left(\dfrac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m. Find the point on the curve $y = x^2 - 7x + 12$, where the tangent is parallel to *x*-axis. *******

7. Discuss applicability Rolle's Theorem for the function f(x) = cosx + sinx in [0,2 $\pi$ ] and hence find a point at which tangent is parallel to X axis. *******

8. Verify Lagrange's mean value theorem for the function $f(x) = x + \dfrac{1}{x}$ in [1,3]. *******

9. Find the intervals in which f(x) = sinx + cosx , $o \le x \le 2\pi$, is increasing or decreasing.
10. Use differentials to find the approximate value of $\sqrt{25.2}$*******
11. Find the interval in which the function given by f(x)= (4sinx – 2x – x cosx) / (2+cos x) is increasing. *******

12. Find the local maximum & local minimum value of function $x^3 - 12x^2 + 36x - 4$ *******
13. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin. *******
14. Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles if $k^2 = 8$.

15. Find the interval in which the function $f(x)= 2x^3-9x^2-24x-5$ is Increasing or decreasing.

16. Find the interval in which the function $f(x) = \sin^4 x + \cos^4 x, 0 \le x \le \dfrac{\pi}{2}$ is increasing or decreasing. *******

17. 6Prove that the curves $x = y^2$ and $xy = k$ cut at right angle if $8k^2 = 1$. *******
18. If *f*(x) = $3x^2 + 15x + 5$, then find the approximate value of *f*(3.02), using diffrentials.*******
19. Find the local maximum and minimum values of function: *f*(x) = sin 2x – x,-π/2 < x < π/2*******

20. Find the interval in which f(x) =sin 3x is increasing or decreasing in [0, π/2].

21. A ladder 5m. long is leaning against a wall . The bottom of the ladder is pulled along the ground, away from the wall at the rate of 2 m./sec. How fast is its height decreasing on the wall,when the foot of the ladder is 4 m away from the wall. *******

22. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$, is strictly decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$*******

23. Find the equation of the tangents to the curve y = √3x-2 which is parallel to the line 4x-2y+5=0. *******

24. Find the intervals in which the function f(x) = $x^3+1/x^3$ increasing or decreasing. *******

25. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4,2]$ *******

26. Find the equation of the tangent and normal to the parabola $y^2 = 4ax$ at ( $at^2$, 2at). *******

27. Using LMVT, find a point on the parabola y = $( x – 3 )^2$ , where the tangent is parallel to the chord (3,0) and (4,1). *******

28. Verify Rolle's theorem for f (x) = $x^3 - 2x^2 - x + 3$ in [0,1] . *******

29. Find the intervals in which f(x) = $x^3 + 2x^2 - 1$ decreasing or increasing

30. It is given for the function of defines by $f(x) = x^3 + bx^2 + ax, x \in [1,3]$ Rolle' theorem helds with $C = 2\frac{1}{\sqrt{3}}$*******

31. Find the intervals in which $f(x) = -2x^3 + 15x^2 - 36x + 1$ is increasing or decreasing. *******

32. Show that the parabola $y^2 = 4x + 4$ & $y^2 = 4 - 4x$ intersect at right angle. *******

33. Find the interval in which the function f is given by $f(x) = Sinx - Cosx, \quad 0 \le x \le 2\pi$ (i) Increasing (ii) Decreasing. *******

35. It is given that for the function f given by $f(x) = x^3 + bx^2 + ax, \quad x \in [1,3]$ Rolle's theorem hold $c = 2 + \frac{1}{\sqrt{3}}$.Find the values of a and b*******

36. Find the equation of the tangent and normal to the curve: $x = aCost + at\,Sint, \; y = aSint - at\,Cost$ at any point 't'. Also show that the normal to the Curve is at a constants distance from the origin.

1. Using differentiate find approximate value of $\sqrt{51}$ .*******

2. The surface area of a spherical bubble is increasing at the rate of $2m^2$/sec. Find the rate of at which the volume of the bubble is increasing at the instant its radius is 6cm. *******
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3. Prove that x/a + y/b = 1 is a tangent to the curve $y = be^{-x/a}$ at the point where the curve cuts y-axis. *******
4. Find the equation of the tangent to the curve $x^2 + 3y - 3 = 0$, which is perpendicular to the line $y = 4x - 5$. ******
5. Find the approximate change in the volume V of cube of side x mts caused by increasing the side 2 %.
6. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing and decreasing. Also find the points on which the tangents are || to the x-axis.
7. Find the equations of the tangent and normal to the curve $16x^2 + 9y^2 = 144$ at $(2, y_1 > 0)$. Also find the point of intersection where both tangent and normal meet the x-axis.
8. A particle moves along $3y = 2x^3 + 3$. Find the points on the curve at which the y-coordinate changes twice as fast as x-coordinate. *******
9. Find the point on the parabola $y = (x - 3)^2$ where the tangent is parallel to chord joining the points (3, 0) and (4, 1). *******
10. The volume of a sphere is increasing at $3cm^3$/ s. what will be the rate at which the radius increases when radius is 2 cm
11. Verify Rolle's theorem for the function $f(x) = x^2-5x+6$ in the interval [2, 3]. *******
12. Water is leaking from a conical funnel at the rate of $5cm^3$/Sec. If the radius of the base of funnel is 5 cm and height 10cm find the rate at which is water level dropping when it is 2.5 cm from the top. *******
13. The length x of a rectangle is decreasing at the rate of 2cm/s and the width y is increasing at the rate of 2cm/s. when x=12 cm and y=5cm, find the rate of change of (a) the perimeter and (b) the area of the rectangle.
14. Using differential, find the appropriate value of $\sqrt[3]{29}$ *******
15. Sand is being poured at the rate of 0.3 $m^3$/sec into a conical pile. If the height of the conical pile is thrice the radius of the base, Find the rate of change of height when the pile is 5cm high. *******
16. Verify the condition of Mean Value Theorem and find a point c in the interval as stated by the MVT for the function given by $f(x) = \log_e x$ on [1, 2]. *******
17. The two equal sides of isosceles triangle with a fixed base 'b' are increasing at the rate of 3cm/sec. How fast is the area decreasing when the two equal sides are equal to the base? *******
18. Verify Rolle's Theorem for the function f(x) = Sin x – Cos x in the interval $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$*******

19. The radius of a balloon is increasing at the rate of 10 cm/sec. At what rate is the volume of the balloon increasing when the radius is 10cm?
20. Find the interval in which the function $f(x) = x^3 - 6x^2 - 36x + 2$
21. Find the intervals in which the following function is increasing : $f(x)=x^4 - 2x^2$. *******
22. Using Rolles theorem, find the points on the curve $y = 16 - x^2$ , $x \in [-1,1]$ where the tangent is parallel to x-axis.
23. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$, is strictly decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$*******
24. Find the equation of the tangent to the curve $x^2 + 3y = 3$ which is parallel to the line $y - 4x + 5 = 0$.
25. A man 160 cm tall; walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1 m/s. How fast is the length of his shadow increasing when he is 1 m away from the pole? *******
26. A point source of light along a straight road is at a height of 'a' metres. A boy 'b' metres in height is walking along the road. How fast is his shadow increasing if he is walking away from the light at the rate of 'c' m/min? *******
27. At what points of the ellipse $16x^2 + 9y^2 = 400$ does the ordinate decrease at the same rate at which the abscissa increases? *******
28. The bottom of a rectangular swimming pool is 25 m by 40 m. Water is pumped out into the tank at the rate of 500 $m^3$/min. Find the rate at which the level of the water in the tank rising. *******
29. An inverted cone has a depth of 40 cm and base of radius 5 cm. Water is poured into it at a rate of 1.5 $cm^3$/ min. Find the rate at which the level of water in the cone is rising when the depth is 4 cm.
30. Water is dripping through a tiny whole at the vertex in the bottom of a conical funnel at a uniform rate of 4 $cm^3$ / s. When the slant height of the water is 3 cm, find the rate of decrease of the slant height of the water, given that the vertical angle of the funnel is $120^0$. *******
31. Oil is leaking at the rate of 16 mL / s from a vertically kept cylindrical drum containing oil. It the radius of the drum is 7 cm and its height is 60 cm, find the rate at which the level of the oil is changing when the oil level is 18 cm. *******

**6MARKS**

1. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12cm is 16cm.
2. Find the equation of line through the point (3, 4) which cuts the 1st quadrant a triangle of minimum area. *******
3. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base. *******
4. Show that the semivertical angle of a cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$. *******
5. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum. *******
6. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening. *******
7. An open box with a square base is to be made out of a given iron sheet of area 27 sq.m. Show that the maximum volume of the box is 13.5 $m^3$.******
8. A point on hypotenuse of right angled triangle is at a distance 'a' and 'b' from the sides

   Show that the length of hypotenuse is at least $(a^{2/3} + b^{2/3})^{3/2}$*******
9. A square piece of tin of side 48 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off, so that the volume of the box is the maximum possible? Also find the maximum volume. *******
10. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere. *******
11. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}(1/3)$. *******

12. *A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 'p' c.m. Show that the window will allow maximum possible light only when the radius of the semi-circle is* $\frac{p}{\pi+4}$ *c.m* *******

12 A rectangle is inscribed in a semi circle or radius a with one of its sides on the diameter of the semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area . *******

13 Show that the surface Area of closed cuboid with square base & given volume is maximum when it is a cube. *******

14 .Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}$cm if $500\pi cm^3$. *******

16. A large window has the shape of a rectangle surmounted by an equilateral triangle. If the

perimeter of the window is 12m find the dimensions of the rectangle that will produce the largest area of the window. *******

17. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone. *******
18. A wire of length 36cm is cut into two pieces. One of them is turned into square and other is into equilateral triangle. Find the length of each piece so that the sum of the areas of two is minimum*******
19. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$*******
20. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle. *******
21. If the lengths of three sides of trapezium other than base are equal to 10 cm, then find the area of the trapezium when it is maximum. *******
22. Given the sum of the perimeter of a square and a circle, show that the sum of their areas is least when the side of the square is equal to diameter of circle.
23. Find a point on the parabola $y^2 = 4x$ which is nearest to the point (2,-8) *******
24. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. *******
25. A given quantity of metal is to be cast into a solid half circular cylinder( i.e. with rectangular base and semicircular ends).Show that the surface area may be minimum, if the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : \pi + 2$. *******

26. A window is in the form of a rectangle surmounted by a semi circular opening . The total perimeter of the window is 10m. Find dimensions of the window to admit maximum light through the whole opening.
27. An open box with a square base is to be made out of a given quantity of sheet of area $a^2$. Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.*******
28. An open tank with square base and vertical sides is to be constructed from a metal sheet so as to hold given quantity of water. Show that the cost of the material will be least when the depth of the tank is half of its width. *******
29. Show that the height of a closed circular cylinder of given total surface are and maximum volume is equal to the diameter its base*******
30. A Cylindrical container with a capacity of 20 cubic feet is to be produced. The top and bottom of the container are to be made of a material that costs Rs.6 per squares foot while the side of the container is made of material costing Rs.3 per squares foot. Find the dimension that will minimize the total cost. *******
31. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle $30^0$ is $\frac{4}{81}\pi h^3$*******
32. Find the largest possible area of a right angled triangle whose hypotenuse is 5cm long. *******
33. Find two positive numbers whose sum is 16 and sum of whose cube is minimum.
34. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius *a* is square of side $\sqrt{2}$ a. *******
35. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank cost Rs. 70 per square meters for the base and Rs. 45 per square meters for sides. What is the cost of least expensive tank? *******
36. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle. *******
37. Prove that : $\frac{x}{1+x} < \log(1+x) < x$ for $x > 0$. *******
38. Prove that : $\tan x > x$ for $x \in (0, \frac{\pi}{2})$.*******
39. Find the intervals on which the following functions are (a) strictly increasing and (b) strictly decreasing: $f(x) = (x+2)e^{-x}$.
40. $f(x) = x^4 - 4x^3 + 4x^2 + 15$.