Name:

XII

ARK MATH ONLINE CLASSES

MATHEMATICS

Section D – 6 Mark Questions

RELATIONS AND FUNCTIONS

1. Let R be the relation on N x N, defined by $(a,\ b)\ R\ (c,\ d) \Leftrightarrow ad(b+c)=bc(a+d)$. Check whether R is an equivalence relation on N x N.
2. Let A be the set of all positive integers and R be a relation on A x A, defined by $(a,\ b)\ R\ (c,\ d) \Leftrightarrow ad=bc$ for all $(a,\ b),\ (c,\ d) \in A\times A$. Show that R is an equivalence relation on A x A.
3. Show that the relation R defined by $(a,\ b)\ R\ (c,\ d) \Rightarrow a+d=b+c$ on A x A, where A= {1, 2, 3, …, 10} is an equivalence relation. Hence write the equivalence class [(3, 4)]; $a,b,c,d \in A$.
4. Show that the given relation R is defined on the set $A=\{x \in \mathrm{Z} : 0 \le x \le 12\}$, given by $R=\{(a,b):|a-b| \text{ is a multiple of } 4\}$, is an equivalence relation. Write the set of all elements related to 1.
5. Show that the relation R in the set A = {1, 2, 3, 4, 5}, given by $R=\{(a,\ b):|a-b| \text{ is divisible by } 2\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other, but no element of {1, 3, 5}is related to any element of {2, 4}.
6. Determine whether the relation r defined on the set R of all real numbers as $R=\{(a,\ b): a,\ b \in R \text{ and } a-b+\sqrt{3} \in S, \text{ where } S \text{ is the set of all irrational numbers}\}$, is reflexive, symmetric and transitive.
7. Let $f: R-\left\{\frac{3}{5}\right\} \to R-\left\{\frac{3}{5}\right\}$ be defined by $f(x)=\dfrac{3x+2}{5x-3}$. Find f^{-1}, if exists.
8. Consider $f: R_+ \to [4,\ \infty)$ given by $f(x)=x^2+4$, show that f is invertible with inverse f^{-1} of f, given by $f^{-1}(x)=\sqrt{x-4}$, where R_+ is the set of all non-negative real numbers.
9. If $f: R \to R$ be defined by $f(x)=x^3-1$, then p[rove that f^{-1} exists and find f^{-1}. Hence find $f^{-1}(26)$ and $f^{-1}(-9)$.
10. Let $f: N \to N$ be a function defined as $f(x)=9x^2+6x-5$, show that $f: N \to S$, where S is the range of f, is invertible. Find the inverse of f and hence $f^{-1}(43)$ and $f^{-1}(163)$.
11. Let $f:[0,\ \infty) \to R$ be a function defined by $f(x)=9x^2+6x-5$. Prove that f is not invertible. Modify, only the co-domain of f to make f is invertible and then find the inverse.
12. Let $f: W \to W$ be defined as $f(n)=\begin{cases} n-1 & \text{if } n \text{ is odd} \\ n+1 & \text{if } n \text{ is even} \end{cases}$. Show that f is invertible and find inverse of f. Here, W is the set of all whole numbers.
13. Let $f(x)=3x^2+7x+10,\ \forall\, x>0$. Show that $f:[0,\ \infty] \to [10,\ \infty]$ is invertible. Also find the inverse of f.

Prepared By: Abdurahiman K. Math Teacher, 0091956 22 111 51, 0091952 68 62 100

14. If $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x, \forall x \in R$, then find $f \circ g$ and $g \circ f$. Hence find $(f \circ g)(-3), (f \circ g)(5)$ and $(g \circ f)(-2)$.

15. Show that if $f : B \to A$ is defined by $f(x) = \dfrac{3x+4}{5x-7}$ and $g : A \to B$ is defined by $g(x) = \dfrac{7x+4}{5x-3}$, then $f \circ g = I_A$ and $g \circ f = I_B$, where $A = R - \left\{\dfrac{3}{5}\right\}$ and $B = R - \left\{\dfrac{7}{5}\right\}$

16. Let $A = R \times R$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a+c, b+d)$. Show that $*$ is commutative and associative. Find the identity element for $*$ on A. Also find the inverse of every element $(a, b) \in A$.

17. Show that the binary operation $*$ on $A = R - \{-1\}$ defined as $a * b = a + b + ab, \forall a, b \in A$ is commutative and associative on A. Also find the identity element of $*$ in A and prove that every element of A is invertible.

18. On the set $\{0, 1, 2, 3, 4, 5, 6\}$, a binary operation $*$ is defined as $a * b = \begin{cases} a+b & if \quad a+b < 7 \\ a+b-7 & if \quad a+b \geq 7 \end{cases}$. Write the operation table of the operation $*$ and prove that 0 is the identity for this operation and each element $a \neq 0$ of the set is invertible with $7-a$ being the inverse of a.

19. Let $A = Q \times Q$, where Q is the set of all rational numbers and $*$ be a binary operation on *A* defined by $(a, b) * (b, d) = (ac, b+ad) \ \ for \ (a, b), (b, d) \in A$. Then find
    i) The identity element of $*$ in *A*.
    ii) Invertible elements of *A*, and hence write the inverse of elements $(5, 3)$ and $\left(\dfrac{1}{2}, 4\right)$.

20. Let $A = Q \times Q$ and $*$ be a binary operation on *A* defined by $(a, b) * (b, d) = (ad + bc, bd) \ \ for \ (a, b), (b, d) \in A$. Is $*$ is commutative and associative? Find the identity element and invertible element in *A* with respect to $*$.

21. A binary operation $*$ is defined on the set R of real numbers by $a * b = \begin{cases} a & if \ b = 0 \\ |a| + b & if \ b \neq 0 \end{cases}$. If at least one of a and b is 0, then prove that $a * b = b * a$. Check whether $*$ is commutative. Find identity element for $*$, if exists.

22. Show that the binary operation $*$ on $A = R - \{-1\}$ defined as $a * b = a + b + ab, \ \forall a, b \in A$ is commutative and associative on *A*. Also find the identity element of $*$ in *A* and prove that every element of *A* is invertible.

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MATRICES AND DETERMINANTS

1. Using matrices, solve the following system of equations
   $x - y + z = 4, \quad 2x + y - 3z = 0, \quad x + y + z = 2$.
2. Using matrices, solve the following system of equations
   $x - y + 2z = 7, \quad 3x + 4y - 5z = -5, \quad 2x - y + 3z = 12$.
3. Using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of equations: $8x + 4y + 3z = 19, \quad 2x + y + z = 5, \quad x + 2y + 2z = 7$.

4. If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 5 & 3\\ 0 & 2 & 1\end{bmatrix}$, find A^{-1} using elementary row transformations.

5. A shopkeeper has 3 varieties of pens 'A','B' and 'C'. Meena purchased 1 pen of each variety for a total of Rs.21.Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of "C" variety for Rs.60. while Shika purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for Rs.70. Using matrix method, find cost of each variety of pen.

6. 10 students were selected from a school on the basis of value for giving awards and were divide into 3 groups The first group comprises hard workers, the second group has honest and law abiding students and third group contains vigilant and obedient student. Double the number of students of the first group added to the number in the second group gives 13,while the combined strength of first and second group is four times that of third group Using matrix method find number of students in each group. Apart from the values; hard work, honesty and respect for law, vigilance and obedience, suggest one more value, which in your opinion, the school should consider for awards.

7. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean . The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added in two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

8. If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$ and $A^3-6A^2+7A+kI_3=o$, find k.

9. Express the matrix $A=\begin{bmatrix}2 & 3 & 1\\ 1 & -1 & 2\\ 4 & 1 & 2\end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

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Integration

1. Evaluate $\int_1^3 \left(x^2+3x+e^x\right)dx$ as the limit of the sum.

2. Evaluate $\int_1^3 \left(e^{2-3x}+x^2+1\right)dx$ as the limit of the sum.

3. Evaluate $\int_0^4 \left(x+e^{2x}\right)dx$ as the limit of the sum.

4. Evaluate $\int_{-2}^2 \left(3x^2-2x+4\right)dx$ as the limit of the sum.

5. Evaluate: $\int_{2}^{5}\left(|x-2|+|x-3|+|x-5|\right)dx$.

6. Evaluate: $\int_{-1}^{2}\left(|x^3-x|\right)dx$.

7. Evaluate: $\int_{0}^{3/2}\left(|x\cos\pi x|\right)dx$.

8. Evaluate: $\int_{0}^{3/2}\left(|x\sin\pi x|\right)dx$.

9. Evaluate: $\int_{-\pi}^{\pi}\left(\frac{2x(1+\sin x)}{1+\cos^2 x}\right)dx$.

10. Show that: $\int_{0}^{\pi/2}\left(\frac{\sin^2 x}{\sin x+\cos x}\right)dx=\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$.

11. Evaluate: $\int_{0}^{\pi}\left(\frac{x\,dx}{1+\sin x}\right)dx$.

12. Evaluate: $\int_{0}^{\pi}\left(\frac{x}{a^2\sin^2 x+b^2\cos^2 x}\right)dx$.

13. Evaluate: $\int_{0}^{\pi/2}\log(\sin x)dx$.

14. Evaluate: $\int_{0}^{\pi}\left(\frac{x\sin x}{1+\cos^2 x}\right)dx$.

15. Evaluate: $\int_{\pi/6}^{\pi/3}\left(\frac{\sin x+\cos x}{\sqrt{\sin 2x}}\right)dx$.

16. Evaluate: $\int_{\pi/6}^{\pi/3}\left(\frac{\sin x+\cos x}{\sqrt{\sin 2x}}\right)dx$.

17. Evaluate: $\int_{0}^{\pi/4}\log(1+\tan x)dx$.

18. Evaluate: $\int_{0}^{\pi}\left(\frac{x\tan x}{\sec x+\tan x}\right)dx$.

19. Evaluate: $\int_{0}^{\pi}\left(\frac{x\tan x}{\sec x\ \mathrm{cosec}\, x}\right)dx$.

20. Evaluate: $\int_{0}^{\pi/4}\left(\frac{\sin x+\cos x}{9+16\sin 2x}\right)dx$.

21. Evaluate: $\int_{\pi/6}^{\pi/3}\left(\frac{1}{1+\sqrt{\cot x}}\right)dx$.

22. Evaluate: $\int_{\pi/4}^{\pi/2}\left(\cos 2x\ \log(\sin x)\right)dx$.

23. Evaluate: $\int_{0}^{\pi/2}\left(\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}\right)dx$.

24. Evaluate: $\int_{-\pi/4}^{\pi/4}\left(\frac{x+\frac{\pi}{4}}{2-\cos 2x}\right)dx$.

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Applications of definite Integrals

1. Using integration, find the area of the region $\left\{(x,\ y): x^2+y^2\le 2ax,\ y^2>ax,\ x,y\ge 0\right\}$.
2. Using integration, find the area of the region $\left\{(x,\ y): x^2+y^2\le 16a^2,\ y^2\le 6ax,\ x,y\ge 0\right\}$.
3. Prove that the curves $y^2=4x$ and $x^2=4y$ divide the area of square bounded by $x=0,\ x=4,\ y=4$ and $y=0$ into three equal parts.
4. Find the area of the region bounded by the two parabolas $y^2=4ax$ and $x^2=4ay$,when $a>0$.

5. Using integration find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2+y^2=4$ at $(1, \sqrt{3})$.
6. Find the area of the region in the first quadrant enclosed by the x-axis, the line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$, using integration.
7. Find the area of the region in the first quadrant enclosed by the y-axis, the line $y=x$ and the circle $x^2+y^2=32$, using integration.
8. Sketch the region bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|$, find its area using integration.
9. Using integration find the area of the region bounded by the curves: $y=|x+1|+1,\ \ x=-3,\ x=3,\ \ y=0$.
10. Using integration find the area of the ΔPQR, coordinates of whose vertices are $P(2,\ 0),\ Q(4,\ 5)$ and $R(6,\ 3)$
11. Using integration, find the area of the region bounded by the lines $2x+y=4,\ 3x-2y=6$ and $x-3y+5=0$.
12. Using integration, find the area of the region bounded by the triangle whose vertices are(-1, 2), (1, 5) and (3, 4).
13. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.
14. Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$, using integration.
15. Find the area of the region enclosed between the two circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$, using integration.
16. Using integration, find the area of the region bounded by the line $x-y+2=0$, the curve $x=\sqrt{y}$ and the y-axis.
17. Sketch the graph of $y=|x+3|$ and evaluate the area under the curve $y=|x+3|$ above the x-axis and between x=-6 and x=0.

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Linear Programming

1. There are two types of fertilizers A and B. A consists of 12% nitrogen and 5% phosphoric acid and B consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 12kg of nitrogen and 12kg of phosphoric acid for her crop. IfA cost Rs 10/kg and B costs Rs 8/kg, then graphically determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?
2. A manufacturer produces nuts and bolts. It takes 2 hours of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 2 hours on machine B to produce a package of bolts. He earns a profit of Rs 24 per package on nuts and Rs 18 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 10 hours a day?
3. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of grinding/cutting machine and a sprayer. It takes 2 hours on the grinding/cutting machine and 3 hours on the

sprayer to manufacture a pedestal lamp. It takes one hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs.25 and that from a shade is Rs.15. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit? Make an L.P.P. and solve it graphically.

4. A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760.00 to invest and has space for at most 20 items. An electronic sewing machine costs him Rs. 360.00 and a manually operated sewing machine Rs. 240.00. He can sell an Electronic Sewing Machine at a profit of Rs. 22.00 and a manually operated sewing machine at a profit of Rs. 18.00. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as a linear programming problem and solve it graphically. Keeping the rural background in mind, justify the 'values' to be promoted for the selection of the manually operated machine.
5. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat is Rs20 and Rs10 respectively, find the number of tennis rackets and crickets bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P and solve graphically.
6. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs50 per kg to purchase Food I and Rs70 per kg to purchase Food II. Formulate this problem as a linear programming problem to minimize the cost of such a mixture, then find the minimum cost.
7. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.
8. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs Rs.10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B costs Rs.4. How many packets of mixed from S and T should the company purchase to honour the contract requirement and yet minimize cost? Make a LPP and solve graphically.
9. In a mid-day meal programme, an NGO wants to provide vitamin rich diet to the students of an MCD school. The dietician of the NGO wishes to mix two types of food in such a way that vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per Kg of vitamin A and 2 units per kg of vitamin C. It

costs Rs 50per kg to purchase Food 1 and Rs 70per kg to purchase Food 2. Formulate the problem as LPP and solve it graphically for the minimum cost of such a mixture?

10. A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society?

11. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

From/To	Cost (in Rs)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? What will be the minimum transportation cost?

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THREE DIMENSIONAL GEOMETRY

1. Find the image of the point having position vector $\left(\hat{i}+3\hat{j}+4\hat{k}\right)$in the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$.
2. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$, which is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.
3. Find the equation of the plane passing through the points A(3, 2, 1), B(4, 2, -2) and C(6, 5, -1) and hence find the value of λ for which A(3, 2, 1), B(4, 2, -2), C(6, 5, -1) and D(λ, 5, 5).
4. Find equation of a plane which passes through the point(3, 2, 0) and contain the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$.
5. Find the vector equation of the plane through the points (2, 1, -1) and (-1, 3, 4) and perpendicular to the plane $x-2y+4z=10$.
6. Find the position vector of foot of perpendicular and perpendicular distance from the point P with position vector $\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ to the plane $\vec{r}.\left(2\hat{i}+\hat{j}+3\hat{k}\right)-26=0$. Also find the image of P in the plane.
7. Find the distance of the point $\left(3\hat{i}-2\hat{j}+\hat{k}\right)$ from the plane $3x+y-z+2=0$ measured parallel to the plane $\frac{x-3}{2}=\frac{y+2}{-3}=\frac{z-1}{1}$. Also find the foot of the perpendicular from the given point upon the given plane.

8. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)=-1$ and $\vec{r}.\left(\hat{i}+\hat{j}-2\hat{k}\right)=0$, and passing through the point (3, -2, -1). Also, find the angle between the two given planes.
9. Find the equation of the plane passing through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$. Also find the distance of the plane obtained above from the origin.
10. Find the distance of the point (2, 12, 5) from the point of intersection of the lines $\vec{r}=\left(2\hat{i}-4\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-2\hat{j}+\hat{k}\right)=0$.
11. Find the coordinates of the point P where the line through A(3, -4, -5), B(2, -3, 1) crosses the plane, passing through the points (2, 2, 1), (3, 0, 1), (4, -1, 0). Also, find the ratio in which P divides the line segment AB.
12. Find the equation of the plane passing through line of intersection of the plane $\vec{r}.\left(\hat{i}+3\hat{j}\right)-6=0$ and $\vec{r}.\left(3\hat{i}-\hat{j}-4\hat{k}\right)=0$, which is at a unit distance from the origin.
13. Find the vector equation of the plane determined by the points A(3, -1, 2), B(5, 2, 4) and C(-1, -1, 6). Also find the distance of point P(6, 5, 9) from the plane.
14. Find the vector and Cartesian equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$.
15. Find the equation of the plane containing two parallel lines $\frac{x-1}{2}=\frac{y+1}{-2}=\frac{z}{3}$ and $\frac{x}{4}=\frac{y-2}{-2}=\frac{z+1}{6}$. Also, find if the plane thus obtained contains the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{5}$ or not.
16. Find the image P' of the point P having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$. Hence find the length of PP'.
17. Find the vector and Cartesian equation of a plane containing the two lines $\vec{r}=\left(2\hat{i}+\hat{j}-3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+5\hat{k}\right)$ and $\vec{r}=\left(3\hat{i}+3\hat{j}+2\hat{k}\right)+\mu\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$. Also show that the line $\vec{r}=\left(2\hat{i}+5\hat{j}+2\hat{k}\right)+P\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ lies in the plane.

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