

SINGLA TUTORIALS

LUDHIANA

Class XII

Pre board exam

Time 3 hr

Sub: Mathematics

General Instructions:

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Questions 1 – 4 in Section A are very short-answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 marks each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 marks each.
6. Questions 24 – 29 in Section D are long-answer II type questions carrying 6 marks each.

Section A

1. Let $S = \{1, 2, 3\}$. Determine whether the function $f : S \rightarrow S$ Where $f(2) = f(3) = 1$, has inverse
2. Let $*$ be a binary operation on the set Q of a rational number such that $a * b = a + ab$ then find $5 * 7$
3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$
4. Find the angle between two vector \vec{a} and \vec{b} with magnitude $\sqrt{3}$ and 2 respectively having

$$\vec{a} \cdot \vec{b} = \sqrt{6}$$

Section B

5. Find the value of the following:

$$\tan \frac{1}{2} \left(\sin^{-1} \frac{2x}{1+x} + \cos^{-1} \frac{1-y^2}{1+y^2} \right)$$

6. Write the value of $\tan^{-1} \left(2 \sin \left(2 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right)$

7. Find the value of x and y if $\begin{bmatrix} x + 10 & y + 2y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x + 4 & 3 \\ 0 & y - 5 \end{bmatrix}$

8. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec . Find the rate at which its area increases, when side is 10 cm long.

9. Find $\int \frac{dx}{x+4x+8}$.

10. Find the sum of the order and the degree of the following differential equation

$$\frac{dy}{dx} + \sqrt[3]{\frac{dy}{dx}} + (1+x) = 0$$

11. Show that the vector A (-2i+3j+5k) ,B(i+2j+3k) and C (7i -k) are collinear.

12. A couple has 2 children . Find the probability that both are boys , If it is known that (i) one of them is a boy (ii) the older child is a boy.

Section C

13. Using properties of determinants ,prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$.

14. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$, Rolle's theorem holds in $[1, 3]$.with $c = 2 + 1/\sqrt{3}$
Find the value of a and b.

15. Differentiate the function $(\sin x)^x + \sin^{-1} \sqrt{x}$ with respect to x .

16. Determine for what value of X, the function $f(x) = x^3 + \frac{1}{x^3}$ is strictly increasing or strictly decreasing.

17. The standard weight of special purpose bricks is 5 kg and it must contain two basic ingredients A and B . A cost Rs 5 per kg and B cost Rs 8 per kg. Strength consideration dictate that the brick should not more than 4 kg of A and minimum 2Kg of B. Since the demand for the product is likely to be related to the price of the brick, find the minimum cost of bricks satisfying the above conditions. Formulate this situation as an LPP and solve it graphically.

18. Find $\int \frac{2x}{(x+1)(x+2)} dx$

19. Prove that $x^2 - y^2 = C(x^2 + y^2)^2$ is the general solution of the differential equation

$$(x^3 - 3xy^2) dx = (y^3 - 3x^2y) dy , \text{ where } C \text{ is a parameter.}$$

20. Find the value of α , if four points with position vector $3i+6j+9k$, $i+2j+3k$, $2i+3j+k$ and $4i+6j+\alpha k$ are Coplanar.

21. A plane meets the coordinates axes in A,B,C such that the centroid of ABC is the points (α, β, γ) .

Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

22. A man is known to speak the truth 4 out of 5 times. He throw a die and report that it is a six. Find

the probability that it is actually a six.

23. There are 4 cards numbered 1, 3 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the number on the two drawn cards. Find the mean and variance of X.

Section D

24. Consider $f: R \rightarrow (-4/3)$ given by $f(x) = \frac{4x+3}{3x+4}$, Show that f is bijective. Find the inverse of f and hence

find $f^{-1}(0)$ and x such that $f^{-1}(x)=2$

25. Find the value of x,y,z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy $A' = A^{-1}$

26. using properties of integral, evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$.

27. using integration find the area of the the region $(x, y : x^2 + y^2 < 2 \text{ and } x, y^2 > ax, x, y > 0)$

28. Define skew lines. Using vector approach, find the shortest distance between the following two

skew lines $r = (8 + 3\alpha)i - (9 + 16\alpha)j + (10 + 7\alpha)k$ $r = 15i + 29j + 5k + \mu(3i + 8j - 5k)$

29. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius r

cm is $\frac{4r}{3}$ cm. Also show that the maximum volume of cone is $\frac{8}{27}$ of the volume of sphere.