PRACTICE PAPER

MATHEMATICS

CLASS XII Time : 3 Hrs

Max. Marks: 100

General Instructions:

i. ALL questions are compulsory.

ii. This question paper contains 29 questions.

iii. Question 1-4 in Section Aare very short-answer type questions carrying 1 mark each.

iv. Question 5-12 in Section B are short-answer type questions carrying 2 marks each.

*v. Question 13-23 in Section C are long-answer **I** type questions carrying 4 marks each.*

*vi. Question 24-29 in Section D are long-answer **II** type questions carrying 6 marks each.*

SECTION.A

Questions 1 to 4 carry 1 mark each.

1. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

2. Examine, if sin|x| is a continuous function.

3. Write the integrating factor of the differential equation $\sqrt{x}\frac{dy}{dx}+y=e^{-2\sqrt{x}}$

4. If the points with position vectors $10\hat{i}+3\hat{j},\ 12\hat{i}-5\hat{j}\ and\ \lambda\hat{i}+11\hat{j}$ are collinear find value of λ

SECTION.B

Questions 5 to 12 carry 2 marks each.

5. Let * be a binary operation on the set R defined by a*b = a + b + ab, where a, b $\in R$. Solve the equation 2 * (3 * x) = 33

6. Solve the equation $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} - 3\begin{pmatrix} x \\ 2y \end{pmatrix} = \begin{pmatrix} -2 \\ -9 \end{pmatrix}$

7. Evaluate $\int \frac{1+\sin x}{1+\cos x}dx$

OR

Evaluate $\int \tan^{-1} x\,dx$

8. Evaluate $\int_2^8 |x-5|dx$

9. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis

10. If $\vec{a}\ and\ \vec{b}$ are two vectors such that $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}\right|$ then prove that $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$.

11. Three cards are drawn without replacement from a pack of 52 cards. Find the probability that the cards drawn are king, queen and jack.

12. A couple has 2 children. Find the probability that both are boys, if it is known that the older child is a boy.

SECTION.C

Questions 13 to 23 carry 4 marks each.

13. Consider $f: R-\left\{-\frac{4}{3}\right\} \to R-\left\{-\frac{4}{3}\right\}$ *given by* $f(x)=\frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence x if $f^{-1}(x)=2$

OR

Show that the relation R in the set N x N defined by (a, b) R(c, d) if $a^2 + d^2 = b^2 + c^2$ for all a, b, c, d $\in N$, is an equivalence relation.

14. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$

15. Using properties of determinants prove that

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$$

16. If $x\sqrt{1+y}+y\sqrt{1+x}=0, (x\neq y)$, then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

OR

If $y=\cos^{-1}\left(\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right)$, then find $\frac{d^2y}{dx^2}$

17. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then show that $\left(1-x^2\right)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0$

18. Find the equation of tangent to the curve y = cos(x + y), $-2\pi \le x \le 0$, that is parallel to the line x + 2y = 0.

19. Evaluate $\int\left(\log(\log x)+\frac{1}{(\log x)^2}\right)dx$

20. Evaluate $\int_0^{\frac{\pi}{4}}\frac{\sin 2\theta}{\sin^4\theta+\cos^4\theta}d\theta$

21. Solve the differential equation (x – siny)dy + (tany)dx = 0, given that y = 0 when x = 0

OR

Show that the differential equation $\left(x\sin^2\frac{y}{x}-y\right)dx+xdy=0$ is homogeneous. Find the

Particular solution of this differential equation, given that $y=\frac{\pi}{4}\,when\,x=1$

22. Two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}-5\hat{k}$ *and* $2\hat{i}+2\hat{j}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

23. Find the vector and cartesian equations of line through the point (1, 2, -4) and perpendicular to the lines

$\vec{r}=\left(8\hat{i}-9\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right) and\ \vec{r}=\left(15\hat{i}-29\hat{j}+5\hat{k}\right)+\lambda\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$

SECTION.D

Questions 24 to 29 carry 6 marks each.

24. If A = $\begin{pmatrix}3 & 2 & 1\\ 4 & -1 & 2\\ 7 & 3 & -3\end{pmatrix}$, the find A^{-1} and hence solve the following system of equation

3x + 4y + 7z = 14

2x – y + 3z = 4

x + 2y – 3z = 0

OR

If A = $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$, find the inverse of A using elementary row transformations and hence solve the matrix equation X A = $\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$

25. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone. Also fine its greatest curved surface area.

26. Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point (2, 1) and the lines whose equations are x = 2y and x = 3y - 3

OR

Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}, x^2 + y^2 - 4x = 0$ and the X - axis

27. Find the position vector of foot of the perpendicular and the perpendicular distance from the point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ to the plane $\vec{r}.(2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$. Also find the image of P in the plane.

OR

Find the distance of the point (3, -2, 1) from the plane 3x + y – z + 2 = 0 measured parallel to the line $\frac{x-3}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also find the foot of the perpendicular from the given point upon the given plane.

28. A retired person wants to invest an amount of Rs. 50,000. His broker recommends investing in two types of bonds A and B yielding 10% and 9% return respectively on the invested amount. He decides to invest at least Rs 20,000 in bond A and at least Rs 10, 000 in bond B. He also wants to invest at least as much in bond A as in bond B. Solve this linear programming problem graphically to maximize his returns.

29. Two numbers are selected at random (without replacement) from the first six positive integers. Let x denote the larger of two numbers obtained. Find the probability distribution of the random variable X and hence find the mean and variance of the distribution.

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