PLAY WITH MATH

TEST NO-05 TIME:-3Hrs.

F.M:-100

General instructions:-

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Questions 1-4 in section A are short-answer type questions carrying 1 mark each.
4. Questions 5-12 in section B are short-answer type questions carrying 2 marks each.
5. Questions 13-23 in section C are long-answer type questions carrying 4 marks each.
6. Questions 24-29 in section D are long-answer type questions carrying 6marks each.

Section-A

1. If A is a square matrix of order 3×3 and |A |=4 find |adj(A)|.

Or,

If $A = \begin{bmatrix} 2 & 3 \\ x & 6 \end{bmatrix}$ is a singular matrix. Find the value of x.

2.If a line makes angle 90° ,60° and 30° with the position direction of x,y and z ,find its direction cosines.

3. Find a vector in the direction of vector i-2j that has magnitude 7 units.

4. Write the degree and order of the differential equation

$$Y = x\frac{dy}{dx} + 2\left(1 + \frac{d^2y}{dx^2}\right)^{1/2}$$

Section-B

5. The length x of a rectangle is decreasing at the rate of 3cm/minute while its breadth y is increasing at the rate of 2cm/min .when x =10 cm and y = 6 , find the rate of change of area of rectangle.

6. Find the derivative of $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ w.r.t x.

Or,

Find $\frac{dy}{dx}$ if $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$

7. Show that $A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$ satisfies the matrix equation $A^2 - 3A - 7I = 0$, hence find A^{-1}

8. Find the Cartesian and vector equation of a line passing through the point (3,-7,-4) and parallel to the line $\frac{x}{2} = \frac{y}{-1} = \frac{z+1}{3}$.

9. Verify lagrnage's mean value theorem for the function f(x) =x^2 + 2x +3, x€ [4,6]

10 . Find $\int \frac{3}{\sqrt{5-4x-x^2}}$ dx

Or,

$\int e^x \frac{1+x}{(2+x)^2}$ dx

11. A person wants to invest upto ₹ 75000. For this two types of bonds B_1 and B_2 are available . Bond _B_1 _gives 8%_ interest while bond B_2 yield 9% interest. He decides to invest at least ₹ 20000 in bond B_1 and not more than ₹ 35000 in bond B_2 . He also wants to invest at least as much in bond B_1 as in the bond B_2 . Make it an LPP for Maximizing the interest and formulate the problem .

12. If A and B are two independent events and P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{2}$, find P(A∪B) hence find the P(not A and not B).

Or,

A couple has two children , find the probability that both children are male ,if it is known that at least one of the children is male .

Section –C

13. Prove that $\tan^{-1} \left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}} \right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

14. For what value of **a** and **b,** the function f defined as:

$$F(x) = \begin{cases} 3ax + b\,, if x < 1 \\ 11, if\ x = 1 \\ 5ax - 2b\,, if\ x > 1 \end{cases}$$ is continuous at x=1 .

15. Evaluate $\int \frac{2x+3}{2x^2-3x-2}\, dx$

Or,

Evaluate $\int_0^{\frac{\pi}{2}} logsinx\, dx$

16. Find the intervals in which the following function is strictly increasing or strictly decreasing

F(x) = 20 - 9x + 6x^2- x^3 .

Or,

For the curve $y=4x^2-2x^5$ find all the points which the tangent passes through the origin.

17. If $y = x^{\cos x} + \sin x^{\cos x}$ find $\frac{dy}{dx}$

18. Find the equation of line of the shortest distance between two lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{9}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-z}{5}$.

19. Let $\vec{a}$ and $\vec{b}$ be such vector that $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$ .if $\vec{a} \times \vec{b}$ is a unit vector then find the angle between $\vec{a}$ and $\vec{b}$.

Or,

Find a vector whose magnitude is 3 units and which is perpendicular to the vector $\vec{a}$ and $\vec{b}$ where $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 4k$ and $\vec{b} = 6\hat{\imath} + 5\hat{\jmath} - 2k$.

20. Find $\int \frac{x^2}{(x^2+1)(x^2+4)} \, dx$

21. Two tailors A and B are paid ₹ 225 and ₹ 300 per day respectively for work . A can stitch 9 shirts and 6 pants per day while B can stitch 15 shirts and 6 pants per day. Formulate the above liner programing problem for minimum cost to stitch 90 shirts and 48 pants . if both the tailors agree to charge 25% less daily on an order by a handicappted institute , what value do they demonstrate.

22. Find the probability distribution of number of doublets in three throws of a pair of dice. Hence find the mean of the distribution.

23.In a factory , manufacturing bolts ,machines A,B and C manufacture respectively 25% ,35%, 40%. Of the bolts of their output 5%,4% and 2% respectively are found to be defective bolts . A bolt is drawn at random from the total production and is found to be defective .Find the probability that it is manufactured by machine B

Section-D

24. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ then find A^{-1} and hence solve the following system of equations $3x+4y+7z = 14$, $2x - y + 3z = 4$, $x+2y- 3z = 0$

Or,

If A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, find the inverse of A using elementary row transformations and hence solve the following matrix equation XA =[1 0 1] .

25. Find the co-ordinates of the point where the line through (3,-4,-5) and (2,-3,1) crosses the plane determined by the points (1,1,4) ,(3,-1,2) and (4,1,-2)

Or,

Find the Cartesian and vector equations of the plane passing through the point (-1,3,2) and is perpendicular to each of the planes: X+2y+3z = 5 , 3x+3y+z =0 hence show that the line $\frac{x+1}{5} = \frac{x-4}{4} = \frac{z+1}{-1}$ is parallel to plane thus obtained .

26. Find a particular solution of the differential equation $\frac{dy}{dx}$ + ycotx = 4x cosecx,(x≠0)

Given that y=0 when x =$\frac{\pi}{2}$.

27. Using integration ,find the area of the region bounded by the triangle whose vertices are(-1,0) ,(1,3) and (3,2)

Or,

Find $\int_1^3 (3x^2 + e^{2x})$ dx as limit of a sum.

28. Show that of all the rectangles inscribed in a given circle ,the square has maximum area.

29. let A =R -{1} . If F : A→A is mapping defined by f(x) = $\frac{x-2}{x-1}$, show that F(x) is bijective , find f^{-1}, also find-

(i) x if $f^{-1}(x) = \frac{5}{6}$

(ii) $f^{-1}(2)$