CLASS XII
GUESS PAPER
MATHS

Topic: Matrices & Determinants

Q1. Construct 3x2 matrix If A = $[a_{ij}]$, where a_{ij} = { i+j,if i≥j; i-j if i< j }

Q2. If $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ find the values of x,y,z

Q3.A matrix of order 3 X 3 has a determinant 15. What is the value of |5A|?

Q4. If A= $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix, find x.

Q5. If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI+bA)^n = a^n .I + na^{n-1} bA$ where I is a unit matrix of the order 2 and n is a positive integer.

Q6. If A= $\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} cos\,n\theta & sin\,n\theta \\ -sin\,n\theta & cos\,n\theta \end{bmatrix}$, $n\epsilon N$

Q7. Find x,y,z . If $\begin{bmatrix} x & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & y & 2 \\ 1 & 2 & z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

Q8. If A= $\begin{bmatrix} 0 & -tan\frac{\alpha}{2} \\ tan\frac{\alpha}{2} & 0 \end{bmatrix}$, I is an identity matrix of order 2.

Show that : (I+A) = (I-A)$\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$

Q9. Show that A = $\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $x^2 + 4x - 42 = 0$. Hence find A^{-1}

Q10. If A = $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 = 8A + KI$.

Q11. Find value of x , If matrices A = $\begin{vmatrix} 4 & -3 & 1 \\ -6 & 7 & -4 \\ 1 & -2 & X \end{vmatrix}$ is not invertible.

Q12.If A= $\begin{bmatrix} 0 & x+2 & 2-x \\ 1-2x & 0 & 2x-1 \\ 3x-8 & x-8 & 0 \end{bmatrix}$ is a Skew Symmetric, find x.

Q13.If A= $\begin{bmatrix} a+2 & x-1 & y+2 \\ 2 & b+3 & z-3 \\ 3 & 4 & c+4 \end{bmatrix}$ find a,b,c,x,y,z for which A is skew symmetric.

Q14.Without expanding show that $\begin{vmatrix} c^2b^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

Q15. Using the properties of determinant , prove the following:

(a) $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ac & b^3 \\ 1 & c^2+ba & c^3 \end{vmatrix}$ = $(b-a)(b-c)(c-a)(a^2+b^2+c^2)$

(b) $\begin{vmatrix} 1+x^2-y^2 & 2xy & -2y \\ 2xy & 1-x^2+y^2 & 2x \\ 2y & -2x & 1-x^2-y^2 \end{vmatrix} = (1+x^2+y^2)^3$

(c) $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+px^3 \\ z & z^2 & 1+px^3 \end{vmatrix}$ = $(1+pxyz)(x-y)(y-z)(z-x)$

(d) $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (b+a)^2 \end{vmatrix}$ = $2abc(a+b+c)^3$

(e) $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & a^2+c^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$ = $4a^2b^2c^2$

(f) $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \alpha+\gamma & \alpha+\beta \end{vmatrix}$ = $(\alpha+\beta+\gamma)\ (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

(g) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$ = $2(a+b+c)^3$

(h) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & b+1 & 1 \\ 1 & 1 & c+1 \end{vmatrix}$ = $abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)$= abc+bc+ca+ab.

Q16. Solve the system of equation by matrix method, where x≠ 0, y≠ 0, z≠ 0:

$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10$, $\frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$.

Q17. Solve the Equations by matrix method :2x-y+3z=5 , 3x+2y-z=7, 4x+5y-5z=9

Q18. Find the inverse of the $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and hence solve the system of equations.

x +2y+z= 4 , -x+y+z= 0 , x- 3y+z = 2

Q19. Using elementary transformations, find the inverseof the matrix :$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

TOPIC: INVERSE TRIGONOMETRIC FUNCTIONS

Q1. Find the principal values of : (a) $\text{Sin}^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (c) $\tan^{-1}(\sqrt{3})$ - $\text{Sec}^{-1}(-2)$

Q2. Evaluate: (a) $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ (b) $\cot^{-1}\left(\cot\left(-\frac{\pi}{4}\right)\right)$ (c) $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

(d) tan $\frac{1}{2}\left[\cos^{-1}\frac{\sqrt{5}}{3}\right]$

(e) sin $\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

Short answer type Questions

Q3. Write the following functions into the simplest form :

(a) $\tan^{-1}\frac{1}{\sqrt{x^2-1}}$, $|x| > 1$ (b) $\tan^{-1}\frac{3a^2x-x^3}{a(a^2-3x^2)}$, a>0, $\frac{-a}{\sqrt{3}} \leq x \leq \frac{-a}{\sqrt{3}}$ (c) $\tan^{-1}\sqrt{\frac{1-cosx}{1+cosx}}$, x<$\pi$

(d) $\tan^{-1}\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$, x∈ $\left(\frac{\pi}{2},\pi\right)$.

Q4. Prove that the following:

(a) $\frac{1}{2}\tan^{-1}x = \cos^{-1}\left[\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}\right]^{1/2}$ (b) $\tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$.

(c) $\cos\left[\tan^{-1}\{\sin(\cot^{-1}x)\}\right] = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$. (d) $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$.

(e) $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right] = \frac{2b}{a}$.

Q5. Show that :

a) $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$. b) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

Q6. Show that :

a) $\cos\left[\tan^{-1}\left(-\frac{4}{3}\right)+\sin^{-1}\frac{12}{13}\right] = \frac{63}{65}$. b) $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

Q7. Solve for x: a) $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$. b) $\tan^{-1}(\sqrt{3}) + \cot^{-1}x = \frac{\pi}{2}$

CONTINUITY & DIFFERENTIATION

Q1. Discuss the continuity of the following functions :

(a) F(x) = $\begin{cases} \frac{x^2-4}{x-2} & x \neq 2 \\ 4 & x = 2 \end{cases}$ at x=2

(b) $f(x) = \begin{cases} \frac{3}{2} - x, & \frac{1}{2} < x < 1 \\ 2 & x = 1 \\ \frac{3}{2} + x, & 1 < x < 2 \end{cases}$ $at\ x = 1$

(c)) F(x) = $\begin{cases} \frac{1-\cos 4x}{x^2}, & x \neq 0 \\ 8, & x = 0 \end{cases}$ at x=0

(d) $f(x) = \begin{cases} \frac{\sin x + x \cos x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ at x=0

Q2. For what k, are the following funcyions continuous at the indicated points :

(a F(x) = $\begin{cases} \frac{1-\cos kx}{x \sin x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ at x=0

(b)) F(x) = $\begin{cases} \frac{k \cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$ at x= $\frac{\pi}{2}$

(c) $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-2}, & 0 \leq x \leq 1 \end{cases}$ at x=0

(d) Exmamine the continuity $f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$

Q1. Differentiate w.r.t. x :

(a) $\sin^2\sqrt{x}$

(b) $\frac{x}{\sqrt{a^2+x^2}}$

(c) $\sin^2 x \cdot \cos^3 x$

(d) $\cot \sqrt{\tan x}$

(e) $\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$

(f) $\sin^{-1} x^2$

(g) $\tan^{-1}\frac{1-x}{1+x}$

(h) $\sqrt{4 + \sqrt{4 + \sqrt{4 + x^2}}}$

Q4. y = $(x+\sqrt{x^2+a^2})^n$ Prove that : $\frac{dy}{dx} = \frac{ny}{\sqrt{a^2+x^2}}$

Q5. If y = $\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$ show that : $\frac{dy}{dx} + \sec^2\{\pi/4 - x\} = 0$

Q6. *Find* $\frac{dy}{dx}$ *for the following functions:*

(i) $\sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$

(ii) $\tan^{-1}\frac{\sqrt[3]{x}+\sqrt[3]{a}}{1-\sqrt[3]{x}\,.\sqrt[3]{a}}$

(iii) $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$

(iv) $\cos^{-1}\left(\frac{3\cos x-4\sin x}{5}\right)$

(v) $\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$

(vi) $\sin^{-1}\{x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\}$

(vii) if y= $\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$, prove that $\frac{dy}{dx} = \sqrt{a^2-x^2}$

(viii) If x= a(cos t +t sin t), y= b(sin t − t cos t). find $\frac{d^2y}{dx^2}$,

(ix.) find $\frac{dy}{dx}$ $x^y + y^x = a^b$

(x). If $\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$, then prove that $\frac{dy}{dx} = \frac{y}{x}$

(xi). If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

(xii). Differentiate $\sin^{-1}\{2x\sqrt{1-x^2}\}$ w.r.t. $\cos^{-1}\frac{1-x^2}{1+x^2}$

(xiii) If $y=e^{a\sin^{-1}x}$, then prove that : $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$

(xiv) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that: $\frac{dy}{dx} = \frac{1}{(1+x)^2}$

(xv) If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

(xvi) If siny = x sin (a+y), prove that : $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

(xvii) If $y\sqrt{1-x^2}$ $+x\sqrt{1-y^2}$ = 1, prove that: $\frac{dy}{dx}$ $=-\sqrt{\frac{1-y^2}{1-x^2}}$

(xviii). find $\frac{dy}{dx}$, y = $x^{\sin x}$ + $(\sin x)^{\cos x}$

Q7. If $e^y + e^x = e^{x+y}$, prove that $\frac{dy}{dx}$ = - e^{y-x}

Q8. If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx}$ = $\frac{y}{x}$

Q9. If x = $\sqrt{a^{\sin^{-1}t}}$, y = $\sqrt{a^{\cos^{-1}t}}$, prove that $\frac{dy}{dx}$ = - $\frac{y}{x}$

Applications of Differentiation

1. If a line y = x + 1 is a tangent to the curve $y^2= 4x$,find the point the of contact ?
2. Find the point on the curve $y = 2x^2– 6x – 4$ at which the tangent is parallel to the x – axis
3. Find the slope of tangent for y = tan x + sec x at x = π/4
4. Show that the function f(x) == $x^3– 6x^2 +12x -99$ is increasing for all x.
5. Find a, for which f(x) = a(x+sinx)+a is increasing
6. Find the intervals in which the function f (x) = $x^3 - 6x^2 + 9x + 15$ is (i) increasing (ii) decreasing.
7. Find the equation of the tangent line to the curve x = θ + sinθ, y = 1+cosθ a=π/4
8. Prove that curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles If $c^4 = 32 a^4$
9. Discuss applicability Rolle's Theorem for the function f(x) = cosx + sinx in [0,2π] and hence find a point at which tangent is parallel to X axis.
10. Verify Lagrange's mean value theorem for the function f(x) = x + 1/x in [1,3].
11. Find the intervals in which f(x) = sin x + cos x , o ≤ x ≤ 2 π, is increasing or decreasing.
12. Find the interval in which the function given by f(x)= $\frac{(4\sin x - 2x - x\cos x)}{(2+\cos x)}$ is increasing.
13. Find the local maximum & local minimum value of function $x^3– 12x^2 + 36x – 4$
14. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.
15. Find the interval in which the function f(x)= $2x^3 -9x^2 -24x-5$ is Increasing or decreasing.
16. Find the equation of the tangents to the curve y = √3x-2 which is parallel to the line 4x-2y+5=0.
17. Find the interval in which the function f is given by f(x)=sin x – cos x ,o ≤ x ≤ 2 π
    (i) Increasing (ii) Decreasing.
18. Find the equation of the tangent to the curve $x^2 + 3y – 3 = 0$, which is perpendicular to the line y = 4x – 5.
19. Find the eq^ of of the tangent and normal to the curve x= a $\sin^3 t$, y= b $\cos^3 t$ at the point t= π/4

20. It is given for the function $f(x)=x^3+bx^2-ax$, $x \in [1,3]$ Rolle' theorem helds with $c = 2 + \frac{1}{\sqrt{3}}$ find a, b.

Type 2

1. Find the intervals on which the following functions are (a) strictly increasing (b) strictly decreasing:

(i) $f(x) = (x+2)e^{-x}$. (ii) $f(x) = x^4 - 4x^3 + 4x^2 + 15$

2. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is square of side $\sqrt{2}$ a.

3. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle 30^0 is $\frac{4}{81}\pi h^3$.

4. A window is in the form of a rectangle surmounted by a semi circular opening . The total perimeter of the window is 10m. Find dimensions of the window to admit maximum light through the whole opening.

5. An open box with a square base is to be made out of a given quantity of sheet of area a^2 . Show that the maximum volume of the box is $\frac{x^2}{6\sqrt{3}}$.

6. An open tank with square base and vertical sides is to be constructed from a metal sheet so as to hold given quantity of water. Show that the cost of the material will be least when the depth of the tank is half of its width.

7. Show that the height of a closed circular cylinder of given total surface area and maximum volume is equal to the diameter its base.

8. Show that the volume of the largest cone that can be inscribed in sphere of radius R is $\frac{8}{27}$ of the volume of the sphere

9. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

10. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.

11. A wire of length 36cm is cut into two pieces. One of them is turned into square and other is into equilateral triangle. Find the length of each piece so that the sum of the areas of two is minimum

12. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$

13. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

14. Show that the semivertical angle of a cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$.

15. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum.

16. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\text{Sin}^{-1}(1/3)$

Practice Assignment

Solve the following Integrals

1. $\int \frac{1}{x(x^7+1)}$ dx.
2. $\int \frac{x^3 \sin(\tan^{-1} x^4)}{(x^8+1)}$ dx.
3. $\int \cos^4 x$ dx.
4. $\int \frac{\sin x}{\sin(x+a)}$ dx.
5. $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+a)}}$
6. $\int \frac{x^2+1}{(x^4+1)}$ dx.
7. $\int \sqrt{\tan x} + \sqrt{\cot x}$ dx.
8. $\int \frac{1}{3+\sin^2 x}$ dx.
9. $\int \sin x . \sin 2x . \sin 3x \, dx$
10. $\int \sin^3 x . \cos^3 x \, dx$
11. $\int \frac{dx}{\cos(x-a)\cos(x-b)}$
12. $\int \frac{e^{2x}-1}{e^{2x}+1}$ dx

13. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx.$

14. $\int \frac{3x+5}{(x^3-x^2-x+1)}dx$

15. $\int \frac{2x+1}{\sqrt{x^2+2x-1}}dx$

16. $\int \frac{5\cos x+6}{2\cos x+\sin x+3}dx.$

17. $\int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx.$

18. $\int \frac{dx}{(1+3e^x-2e^{2x})}$

19. $\int \frac{1}{(x^4+1)}dx$

20. $\int \frac{x^2}{(x^4+1)}dx$

21. $\int \frac{dx}{\sqrt{15-8x^2}}$

22. $\int \sqrt{\tan x}\ dx.$

23. $\int \frac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}dx.$

24. $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

25. $\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$

26. $\int \frac{\sin x}{\sin(3x)}dx$

27. $\int \frac{2\sin 2x-\cos x}{6-\cos^2 x-\sin x}dx.$

28. $\int \frac{6x+5}{\sqrt{6+x-2x^2}}dx.$

29. $\int \frac{\sqrt{\tan x}}{\sin(x)\cos x}dx$

30. $\int \frac{(x+1)e^x}{\cos^2(xe^x)}dx.$

31. $\int \frac{dx}{\cos^2 x\,(1-\tan x)^2}$.

32. $\int \frac{e^x\log(\sin e^x)}{\tan e^x}dx.$

33. $\int \frac{\cos 2x.\sin 2x}{\sqrt{9-\cos^4 2x}}dx$

34. $\int \frac{dx}{e^{-x}+e^x}$

35. $\int \frac{x^{1/2}}{\sqrt{x^3-a^3}}dx$

36. $\int \sqrt{\frac{a-x}{a+x}}dx$

37. $\int \frac{dx}{\sqrt{(x-1)(x-2)}}$

38. $\int \frac{dx}{\sqrt{(8+3x-x^2)}}$

39. $\int \frac{8}{(x+2)(x^2+4)}dx$

40. $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

41. $\int \frac{\cos x}{(1-\sin x)^3(2+\sin x)}dx$

42. $\int \frac{3x+5}{(x^3-x^2-x+1)}dx$

42. $\int \frac{5x}{(x^2+9)(x+1)}dx$

43. $\int \frac{(2+\tan^2\theta)\sec^2\theta}{(1+\tan^3\theta)}d\theta$

44. $\int \frac{x^4+1}{(x^6+1)}dx$

45. $\int \frac{\sec^2\theta}{(\tan^3\theta+4\tan\theta)}d\theta$

46. $\int \frac{e^x\log(e^x)}{e^x}dx.$

47. $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx.$

48. $\int e^x(\tan x-\log\cos x)dx.$

49. $\int e^x\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx.$

50. $\int e^{\tan^{-1}x}\left(\frac{1+x^2+x^4}{1+x^2}\right)dx.$

51. $\int \frac{\log x-1}{(\log x)^2}dx.$

52. $\int e^x(\cos 3x)dx.$

53. $\int e^{Ax}(\mathrm{Sin}\,Bx)dx.$

54. $\int \cos^{-1}x\,dx.$

55. $\int \tan^{-1}\sqrt{x}\,dx.$

56. $\int x\tan^{-1}x^2\,dx.$

57. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx.$ 58. $\int x^2 \tan^{-1} x\,dx.$ 59. $\int x^2 \operatorname{cscec}^{-1} x\,dx.$

60. $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\,dx.$ 61. $\int \log\left(x+\sqrt{a^2+x^2}\right)dx.$ 62. $\int \frac{\sqrt{16+(\log x)^2}}{x}\,dx.$

63. $\int \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\,dx.$ 64. $\int \sqrt{2x^2+3x+4}\,dx.$ 65. $\int \sqrt{3-2x-2x^2}\,dx.$

66. $\int (3x-2)\sqrt{x^2+x+1}\,dx.$ 67. $\int \frac{1+\operatorname{Sin} x}{\operatorname{Sin}x\,(1+\operatorname{Cos} x)}\,dx.$ 68. $\int \frac{dx}{(x-1)\sqrt{2x+3}}\,dx.$

69. $\int \sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{x}\,dx.$ 70. $\int \frac{\sqrt{x^2+b^2}}{x}dx.$

CONTD…… INDEFINITE INTEGRATION BYPARTS &PARTIAL FRACTION and DEFINITE INTEGRATION

46. $\int x \sin^{-1} x\, dx$ 47. $\int \frac{\log x}{(x^2)}\,dx$ 48. $\int\left\{\frac{1}{x^2}-\frac{2}{x^3}\right\}e^x\,dx.$ 49. $\int \frac{x^4}{(x-1)(x^2+1)}\,dx$

50. $\int e^x\,(\tan x-\log\cos x)\,dx.$ 51. $\int \frac{\log x-1}{(\log x)^2}\,dx.$ 52. $\int e^x\,(\cos 3x)\,dx.$

53. $\int e^{Ax}\,(\operatorname{Sin} Bx)\,dx.$ 54. $\int \cos^{-1} x\,dx.$ 55. $\int \tan^{-1}\sqrt{x}\,dx.$

56. $\int x \tan^{-1} x^2\,dx.$ 57. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx.$ 58. $\int x^2 \tan^{-1} x\,dx.$

59. $\int x^2 \operatorname{cscec}^{-1} x\,dx.$ 60. $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\,dx.$ 61. $\int \log\left(x+\sqrt{a^2+x^2}\right)dx.$

62. $\int \frac{\sqrt{16+(\log x)^2}}{x}\,dx.$ 63. $\int \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\,dx.$ 64. $\int \sqrt{2x^2+3x+4}\,dx.$

65. $\int \sqrt{3-2x-2x^2}\,dx.$ 66. $\int (3x-2)\sqrt{x^2+x+1}\,dx.$ 67. $\int \frac{1+\operatorname{Sin} x}{\operatorname{Sin}x\,(1+\operatorname{Cos} x)}\,dx.$

68. $\int \frac{dx}{(x-1)\sqrt{2x+3}}\,dx.$ 69. $\int \sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{x}\,dx.$ 70. $\int \frac{\sqrt{x^2+b^2}}{x}dx.$

71. $\int e^x\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx.$ 72. $\int e^{\tan^{-1}x}\left(\frac{1+x^2+x}{1+x^2}\right)dx.$ 73. $\int e^{2x}\left(\frac{\sin 4x-2}{1-\cos 4x}\right)dx.$

74. $\int \frac{5x}{(x^2+9)(x+1)}\,dx$ 75. $\int \frac{\cos x}{(1-\sin x)^2(2+\sin x)}\,dx$

Evaluate the integrals as limit of sums. (Q1 to Q4)

Q1. $\int_1^2 (5+2x)\,dx$ Q2 $\int_1^4 (x^2+2)\,dx$ Q3. $\int_1^3 (3x^2+x)\,dx$ Q4. $\int_2^4 (2x^2+3x+1)\,dx$

Q5. $\int_{-1}^1 log(\frac{4-x}{4+x})\,dx$. Q6. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^3x\,dx$. Q7. $\int_0^3 |x-1|+|x-2|+|x|\,dx$

Q8. $\int_{-3}^3 |x+1|\,dx$.

9. $\int_0^{\pi/4} \sqrt{1+sin2x}\,dx$. 10. $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx$ 11. $\int_0^{\frac{\pi}{2}} \frac{sin^7x}{cos^7x+sin^7x}dx$ 12. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}}$.

13. $\int_{\pi/6}^{\pi/3} \frac{sinx+cosx\ dx}{\sqrt{sin\,2x}}$. 14. . $\int_0^{\pi} \frac{x\,sinx}{1+cos^2x}\,dx$ 15. $\int_0^{\pi} \frac{x\,tanx}{secx+tanx}\,dx$

16. Prove that: $\int_0^1 \tan^{-1}(\frac{1}{1-x+x^2})\,dx = \frac{\pi}{2} - \log 2$

17. $\int_{-\pi/2}^{\pi/2} (sin|x| - cos|x|)\,dx$. 18. $\int_0^{\pi} \frac{dx}{a^2cos^2x+b^2sin^2x}$ 19. $\int_0^{\pi/4} log(1+tanx)\,dx$ 20. $\int_0^{\pi} \frac{x\,tanx\,dx}{secx\,cosecx}$

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CONTD…… INDEFINITE INTEGRATION BYPARTS &PARTIAL FRACTION and DEFINITE INTEGRATION

46. $\int x\ sin^{-1}x\ dx$ 47. $\int \frac{log\,x}{(x^2)}\,dx$ 48. $\int\{\frac{1}{x^2}-\frac{2}{x^3}\}e^x\,dx$. 49. $\int \frac{x^4}{(x-1)(x^2+1)}\,dx$

50. $\int e^x\ (\tan x - \log\cos x)dx$. 51. $\int \frac{\log x-1}{(\log x)^2}dx$. 52. $\int e^x\ (\cos 3x)dx$.

53. $\int e^{Ax}\ (\sin Bx)dx$. 54. $\int \cos^{-1}x\,dx$. 55. $\int \tan^{-1}\sqrt{x}\,dx$.

56. $\int x\ \tan^{-1}x^2\,dx$. 57. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$. 58. $\int x^2\ \tan^{-1}x\,dx$.

59. $\int x^2\ cscec^{-1}x\ dx$. 60. $\int \frac{\sin^{-1}x}{(1-x^2)^{3/2}}\,dx$. 61. $\int \log(x+\sqrt{a^2+x^2})\,dx$.

62. $\int \frac{\sqrt{16+(\log x)^2}}{x}\,dx$. 63. $\int \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\quad dx$.

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64. $\int \sqrt{2x^2+3x+4}\,dx.$ 65. $\int \sqrt{3-2x-2x^2}\,dx.$ 66. $\int (3x-2)\sqrt{x^2+x+1}\,dx.$ 67. $\int \frac{1+\sin x}{\sin x\,(1+\cos x)}\,dx.$ 68. $\int \frac{dx}{(x-1)\sqrt{2x+3}}\,dx.$

69. $\int \sqrt{\frac{1-x}{1+x}} \cdot \frac{1}{x}\,dx.$ 70. $\int \frac{\sqrt{x^2+b^2}}{x} dx.$ 71. $\int e^x \left(\frac{1+\sin 2x}{1+\cos 2x}\right) dx.$

72. $\int e^{\tan^{-1} x} \left(\frac{1+x^2+x}{1+x^2}\right) dx.$ 73. $\int e^{2x} \left(\frac{\sin 4x-2}{1-\cos 4x}\right) dx.$ 74. $\int \frac{5x}{(x^2+9)(x+1)}\,dx$

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Evaluate the integrals as limit of sums. (Q1 to Q4)

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20. $\int_0^{\pi} \frac{x \tan x\, dx}{\sec x\, \operatorname{cosec} x}$

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AREA BETWEEN THE CUVES (USING INTEGRATION)

Q1.Find the area bounded by the curve $x^2 = 4y$ and straight line $x = 4y - 2$

Q2.Find the area of the region bounded by $y = x^2 + 2$, $y = x$, $x = 0$, and $x = 3$.

Q3. Find the area of the circle : $x^2 + y^2 = 4$ which is exterior to the parabola $y^2 = 6x$.

Q4.Find the area enclosed between the circles: $x^2 + y^2 = 4$, $(x-2)^2 + y^2 = 4$.

Q5. Find the area enclosed between the parabolas: $x^2 = 4x$, $y^2 = 4x$.

Q6.Find the area of the triangle formed by the points (2,1), (4,3),and (5,2).

Q7.Find the area of the parabola: $y^2 = 4a^2 (x-3)$ above x-axis under the constraints x = 3, y= 4a .

Q8.Draw the rough sketch of (a) { $x^2 \leq y \leq |x|$ } and (b) {(x,y) : $|x-1| \leq y \leq \sqrt{5-x^2}$ }.find the area of the region enclosed.

DIFFERENTIAL EQUATION:

Q1. Show that , y =cosx, is the solution of differential equation , $\frac{dy}{dx} + \sin x = 0$

Q2 Show that $xy = a e^{x} + b e^{-x} + x^2$ is a solution of differential equation : $x\{d^2y/dx^2\} + 2\{dy/dx\} - xy + x^2 - 2$

Q3. Solve the following differential equations :

i) $\frac{dy}{dx} = \sin(x+y)$

ii) $y\,dx - (x+2y^2)dy = 0$

iii) $\cos x(1+\cos y)\,dx - \sin y(1+\sin x)\,dy = 0$

iv) $\log \frac{dy}{dx} = ax+by$.

v) $\frac{dy}{dx} - 3y\cot x = \sin 2x$, {y=2when x= $\frac{\pi}{2}$}

vi) $\frac{dy}{dx} + 2y = x\,e^{4x}$

vii) $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

viii) $x\frac{dy}{dx} - y = x\tan\frac{y}{x}$, { y= $\frac{\pi}{2}$ when x=1}

ix) $(1+y^2)dx = (\tan^{-1} y - x)\,dy$, given y (0) =0

x) $\sin^{-1}\frac{dy}{dx} = x+y$.

xi) $\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$

xii) $\frac{dy}{dx} + \frac{x^2+3y^2}{3x^2+y^2} = 0$

xiii) $(1+e^{x/y})\,dx + e^{x/y}\{1 - \frac{x}{y}\}\,dy = 0$

xiv) $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0$; y=0 when x=1

xv) $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$

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Edited by : **Satvinder singh Mob : 9729064004 email :** sspawar.singh00@gmail.com