

Name.....

MANISH MATHEMATICS POINT
25488956

Time: 3 Hours

Max Marks: 100

GENERAL INSTRUCTIONS

For Part A

Question numbers 1 to 8 are of 3 marks each
 Questions number 9 to 15 are of 4 marks each
 Questions number 16 to 18 are of 6marks each

For Part C

Question numbers 19 to 22 are of 3 marks each
 Questions number 23 to 25 are of 4 marks each
 Questions number 26 is of 6 marks

1	2	3	4	5	6	7	8	9	10	11	12	13	Total
14	15	16	17	18	19	20	21	22	23	24	25	26	

Part A

1. If $A = \begin{bmatrix} 0 & -1 & 5 \\ 6 & 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 1 & 0 \\ 7 & -6 \end{bmatrix}$, verify that $(AB)' = B'A'$

2. Show that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$

- Three cards are drawn successively with replacement from a well shuffled deck of 52 cards. A random variable X denotes the number of hearts in three cards drawn .Determine the probability distribution of X
- In a class, 30% of students offered Mathematics, 20% offered Chemistry and 10% both If a student is selected at random, find the probability he has offered Mathematics or chemistry.
- Evaluate $\int \sqrt{\sec x - 1} dx$
- Evaluate $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$ OR $\int \tan^4 x dx$
- Solve $\sqrt{1-y^2} dx = (\sin^{-1} y - x) dy$, $y(0) = 0$
- Solve $x \frac{dy}{dx} = y - x \cdot \tan\left(\frac{y}{x}\right)$
- Using the properties of Boolean algebra prove that if $x + y = x + z$ and $x'y = x'z$ then $y = z$

10. Show that $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2} = \frac{1}{4}$

11. Differentiate $\sqrt{\tan x}$ using first principle

12. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

13. A man 2m high walks at a uniform speed of 5 km/h away from a lamppost which is 6m high. Find the rate at which the length of his shadow increases

14. Evaluate: $\int \frac{1}{\cos x (\sin x + 2 \cos x)} dx$

15. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$.

16. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two matrices, find AB and using AB find solution of

$$x - y = 3$$

$$2x + 3y + 4z = 17$$

$$y + 2z = 7$$

17. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and the straight line $\frac{x}{4} + \frac{y}{3} = 1$

18. Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$

PART C

19. A bill of Rs 35000 drawn on April 19, 2002 at 6 months, was discounted at certain date at 5% pa and proceeds were Rs 34300. When was bill discounted

20. Find the face value of the bill discounted at 5% pa, 73 days earlier than date of maturity, the bankers gain being Rs 10

21. Five cards are drawn successively with replacement from the deck of cards, Find the mean and variance of number of spade cards

22. In a test, an examinee either guesses or copies or knows the answer to the multiple choice question with four choices. The probabilities that he copies is $\frac{1}{6}$, that he guess is $\frac{1}{3}$. The probability that his answer is correct given that he copied is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.

23. A and B are partners sharing profits in ratio 4:3 .C is admitted for $\frac{1}{4}$ th share. Find the new profit sharing ratio and sacrificing ratio.
24. A man wishes to collect Rs 4,00,000 for a house at the time of his retirement which is due after 18 years. If the rate of interest is 12% p.a. compounded annually, how much should he deposit annually during the remaining 18 years of his service in order to receive the said amount?[use $(1.12)^{18}=7.6899$]
25. A computer floppy manufacture produces x floppies per hour at a total cost of $\left(\frac{x^2}{25} + 3x + 100\right)$ He is a monopolist and demand for his product is given by $x = 75 - 3p$, where p is price per unit Find the number of floppies produced per hour to maximize his profit also find monopoly price
26. A retired person has Rs 70,000 to invest and two types of bonds are available; first one pays 8% and second one pays 10% pa. But due to certain constrains he has to invest at least Rs 10,000 in first type and atmost 30,000 in second type of bonds .How should he plan his investment to get maximum returns .