TARGET MATHEMATICS
THE EXCELLENCE KEY
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Time: 1 hr 30 min] **[Max Marks 50**

Section A

1 Find the identity element for the binary operation $\square$ defined on $Q-\{0\}$ as $a \square b = \frac{ab}{2} \forall\, a,b \in Q-\{0\}$.

2 Prove that $\cot\left\{\frac{\pi}{4}-2\cot^{-1}3\right\}=7$

3 Find the non-zero values of x satisfying the matrix equation $x\begin{bmatrix}2x & 2\\ 3 & x\end{bmatrix}+2\begin{bmatrix}8 & 5x\\ 4 & 4x\end{bmatrix}=2\begin{bmatrix}(x^2+8) & 24\\ 10 & 6x\end{bmatrix}$ $[Ans: x=4]$

4 Differentiate $\frac{8^x}{x^8}$

Section B

5 Discuss the continuity of the function $f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, & if\ x\neq 0\\ 0, & if\ x=0\end{cases}$ at x = 0.

6 If $x=e^{\theta}\left(\theta+\frac{1}{\theta}\right), y=e^{-\theta}\left(\theta-\frac{1}{\theta}\right)$ then find $\frac{dy}{dx}$.

7 Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{77}{85}$

8 If $\Delta=\begin{vmatrix}a & p & x\\ b & q & y\\ c & r & z\end{vmatrix}=16$ *then* $\Delta_1=\begin{vmatrix}p+x & a+x & a+p\\ q+y & b+y & b+q\\ r+z & c+z & c+r\end{vmatrix}=32$

9 Prove that the curves $y^2=4x$ and $x^2+y^2-6x+1=0$ touch each other at the point $(1,2)$.

10 Evaluate : $\int\frac{x^{\frac{1}{2}}}{1+x^{\frac{3}{4}}}dx$

11 Evaluate : $\int\frac{\sin^{-1}x}{\left(1-x^2\right)^{\frac{3}{4}}}dx$

Section C

12 Evaluate : $\int_0^{\frac{1}{2}} \frac{dx}{\left(1+x^2\right)\sqrt{1-x^2}}$

13 The sum of surfaces areas of a rectangular parallelepiped with sides x, 2x and $\frac{x}{3}$ and a sphere is given to constant. Prove that the sum of their volume is minimum, if x is equal to three times the radius of the sphere.

14 If $A=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, then find AB and use this to solve the system of equations $y+2z=7, x-y=3$ and $2x+3y+4z=17$.

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