RISE OF NATION ACADEMY

"We Create the Impeccable Creature"

Test Paper

Standard – XIIth

Subject –Mathematics

Topic – Full Course

Date – 03/02/2019 **Time – 03:00 hrs.**

Max. Marks: 100 **Min. Marks: 50**

SECTION.A

Questions 1 to 4 carry 1 mark each.

1. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+sin\theta & 1 \\ 1 & 1 & 1+cos\theta \end{vmatrix}$

2. Examine, if sin |x| is a continuous function.

3. Write the integrating factor of the differential equation $\sqrt{x}\frac{dy}{dx} + y = e^{-2\sqrt{x}}$.

4. If the points with position vectors are collinear find value of $10\hat{\imath} + 3\hat{\jmath}, 12\hat{\imath} - 5\hat{\jmath}$ and $\lambda\hat{\imath} + 11\hat{\jmath}$ **are collinear find value of** λ.

SECTION.B

Questions 5 to 12 carry 2 marks each.

5. Let * be a binary operation on the set R defined by a*b = a + b + ab, where a, b $\in R$
Solve the equation 2 * (3 * x) = 33 R

6. Solve the equation $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} - 3\begin{pmatrix} x \\ 2y \end{pmatrix} = \begin{pmatrix} -2 \\ -9 \end{pmatrix}$.

7. Evaluate $\int \frac{1+sinx}{1+cosx} dx$.

OR

Evaluate $\int \tan^{-1} x \, dx$.

8. Evaluate $\int_8^2 |x-5| dx$.

9. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis

10. If $\vec{a}\ and\ \vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{a}|$ then prove that $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$.

11. Three cards are drawn without replacement from a pack of 52 cards. Find the probability that the cards drawn are king, queen and jack.

12. A couple has 2 children. Find the probability that both are boys, if it is known that the older child is a boy.

SECTION.C

Questions 13 to 23 carry 4 marks each.

13. Consider $f: R-\left\{-\frac{4}{3}\right\} \longrightarrow R-\left\{-\frac{4}{3}\right\}$ $given\ by\ f(x)=\frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence x if $f^{-1}(x) = 2$

OR

Show that the relation R in the set N x N defined by (a, b) R(c, d) if $a^2+d^2=b^2+a^2$ for all a, b, c, d, ∈ N, is an equivalence relation.

14. Show that $\tan\left(\frac{1}{2}sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

15. Using properties of determinants prove that

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx).$$

16. If $x\sqrt{1+y}+y\sqrt{1+x}=0, (x \neq y), then\ prove\ that\ \frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

OR

If $y = cos^{-1}\left(\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right)$, then find $\frac{d^2y}{dx^2}$.

17. If $y = \frac{sin^{-1}x}{\sqrt{1-x^2}}, then\ show\ that\ (1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

18. Find the equation of tangent to the curve y = cos (x + y), -$2\pi \leq x \leq 0$ that is parallel to the line x + 2y = 0.

19. Evaluate $\int \left\{\log(\log x) + \frac{1}{(\log x)^2}\right\} dx$

20. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin 2\theta}{\sin^4\theta + \cos^4\theta} d\theta$

21. Solve the differential equation (x – sin y) dy + (tan y) dx = 0, given that y = 0 when x = 0

OR

Show that the differential equation $\left(x\sin^2\frac{y}{x} - y\right) dx + x dy = 0$, is homogeneous. Find the Particular solution of this differential equation, given that $y = \frac{\pi}{4}$ *when* $x = 1$.

22. Two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ *and* $2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ Find the two-unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

23. Find the vector and cartesian equations of line through the point (1, 2, -4) and perpendicular to the lines

$\vec{r} = (8\hat{\imath} - 9\hat{\jmath} + 10\hat{k}) + \lambda(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$ *and* $\vec{r}$
$= (15\hat{\imath} - 29\hat{\jmath} + 5\hat{k}) + \lambda(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$

SECTION.D

Questions 24 to 29 carry 6 marks each.

24. If $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{pmatrix}$, *the find* A^{-1} and hence solve the following system of equation

3x + 4y + 7z = 14, 2x – y + 3z = 4, x + 2y - 3z = 0.

OR

If $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$, find the inverse of A using elementary row of transformation and hence solve the matric equation X A = (1 0 1)

25. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone. Also fine its greatest curved surface area.

26. Using integration, find the area bounded by the tangent to the curve 4y = x^2 at the point (2, 1) and the lines whose equations are x = 2y and x = 3y - 3

OR

Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}, x^2 + y^2 - 4x = 0$ and the X axis

27. Find the position vector of foot of the perpendicular and the perpendicular distance from the point P with position vector $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ to the plane $\vec{r}.\left(2\hat{\imath} + \hat{\jmath} + 3\hat{k}\right) - 26 = 0$. Also find the image of P in the plane.

OR

Find the distance of the point (3, -2, 1) from the plane 3x + y – z + 2 = 0 measured parallel to the line $\frac{x-3}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$ Also find the foot of the perpendicular from the given point upon the give plane.

28. A retired person wants to invest an amount of Rs. 50,000. His broker recommends investing in two types of bonds A and B yielding 10% and 9% return respectively on the invested amount. He decides to invest at least Rs 20,000 in bond A and at least Rs 10, 000 in bond B. He also wants to invest at least as much in bond A as in bond B. Solve this linear programming problem graphically to maximize his returns.

29. Two numbers are selected at random (without replacement) from the first six positive integers. Let x denote the larger of two numbers obtained. Find the probability distribution of the random variable X and hence find the mean and variance of the distribution.