

Time: 3hrs

mm: 70

General Instructions:

1. All questions are compulsory. There are 27 questions in all.
2. This question paper has four sections: Section A, Section B, Section C and Section D.
3. Section A contains five questions of one mark each(1-5), Section B contains seven questions of two marks each, (6-12)Section C contains twelve questions of three marks each(13-24), and Section D contains three questions of five marks each(25-27).
4. There is no overall choice.

Section A

1. The least count of the main scale of a screw gauge is 1 mm. Find the minimum number of divisions on its circular scale required to measure 5mm diameter of wire? 1

$$\text{Least count} = \frac{\text{Pitch}}{\text{Number of division on circular scale}}$$

$$5 \times 10^{-6} = \frac{10^{-3}}{N}$$

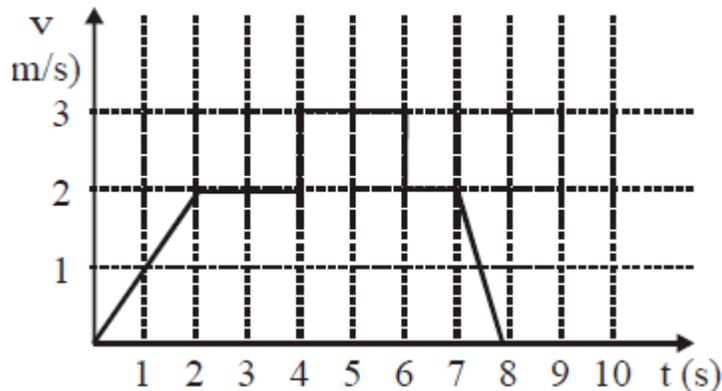
$$N = 200$$

2. The radii of curvature of both the surfaces of a lens are equal. If one of the surfaces is made plane by grinding, how will the focal length and power of the lens change? 1

Focal length gets doubled.
Power is halved.

1/2
1/2

3. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5s$? 1



Ans:

$$S = \text{Area under graph}$$

$$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9 \text{ m}$$

4. If the angular momentum of a planet of mass m , moving around the Sun in a circular orbit is L , about the center of the Sun, find its areal velocity? 1

$$\frac{dA}{dt} = \frac{L}{2m}$$

5. Write the displacement equation representing the following conditions obtained in a simple harmonic motion : Amplitude = 0.01 m, Frequency = 600 Hz, Initial phase = $\frac{\pi}{6}$.

1

Ans. $y = a \sin(2\pi\nu t + \phi_0)$

$$= 0.01 \sin\left(1200\pi t + \frac{\pi}{6}\right)$$

6. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures?

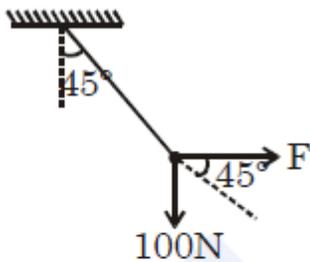
2

$$\frac{\Delta V}{V} = 2 \frac{\Delta d}{d} + \frac{\Delta h}{h} = 2 \left(\frac{0.1}{12.6} \right) + \frac{0.1}{34.2}$$

$$V = 12.6 \times \frac{\pi}{4} \times 314.2$$

7. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, find the magnitude of the force applied is ($g = 10 \text{ ms}^{-2}$)

2



Sol.

at equation

$$\tan 45^\circ = \frac{100}{F}$$

$$F = 100 \text{ N}$$

8. A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} + 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?

2

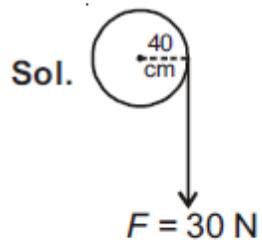
$$\begin{aligned} \text{Work done} &= \vec{F} \cdot \vec{d} \\ &= 12\text{J} \end{aligned}$$

work energy theorem

$$\begin{aligned} w_{\text{net}} &= \Delta \text{K.E.} \\ 12 &= K_f - 3 \\ K_f &= 15\text{J} \end{aligned}$$

9. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N?

2



$$\tau = I \alpha$$

$$F \times R = MR^2 \alpha$$

$$30 \times 0.4 = 3 \times (0.4)^2 \alpha$$

$$12 = 3 \times 0.16 \alpha$$

$$400 = 16 \alpha$$

$$\alpha = 25 \text{ rad/s}^2$$

10. An ideal gas occupies a volume of 2m^3 at a pressure of $3 \times 10^6 \text{ Pa}$. Find the energy of the gas?

2

$$\text{Energy} = \frac{1}{2} nRT = \frac{f}{2} PV$$

$$= \frac{f}{2} (3 \times 10^6)(2)$$

$$= f \times 3 \times 10^6$$

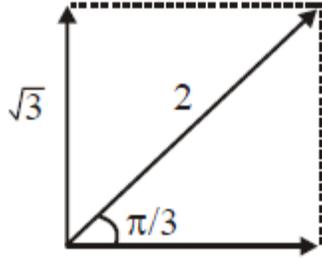
Considering gas is monoatomic i.e. $f = 3$

$$E. = 9 \times 10^6 \text{ J}$$

11. A simple harmonic motion is represented by: $y = 5(\sin 3\omega t + 3 \cos 3\omega t)$ cm. What is the amplitude and time period of the motion?

2

Sol.



$$y = 5 \left[\sin(3\pi t) + \sqrt{3} \cos(3\pi t) \right]$$

$$= 10 \sin \left(3\pi t + \frac{\pi}{3} \right)$$

Amplitude = 10 cm

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = \frac{2}{3} \text{ sec}$$

12. A screen is placed 90 cm away from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens. 2

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

(i) $\therefore \frac{1}{f} = \frac{1}{90-u} - \frac{1}{-u} = \frac{1}{90-u} + \frac{1}{u}$ -----(1) 1/2

(ii) $\frac{1}{f} = \frac{1}{70-u} - \frac{1}{-(u+20)} = \frac{1}{70-u} + \frac{1}{u+20}$ -----(2) 1/2

Solving eqⁿ (1) and (2), u=35 cm 1/2

Using lens formula

$$f = 21.4 \text{ cm}$$

(Alternatively if a candidate calculates the focal length by using the formula $4fD = D^2 - d^2$, award full marks.) 1/2

13. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1 . On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2 . Find the time taken by her to walk up on the moving escalator. 3

Sol. Velocity of girl w.r.t. elevator $= \frac{d}{t_1} = v_{ge}$

Velocity of elevator w.r.t. ground $v_{eG} = \frac{d}{t_2}$ then

velocity of girl w.r.t. ground

$$\vec{v}_{gG} = \vec{v}_{ge} + \vec{v}_{eG}$$

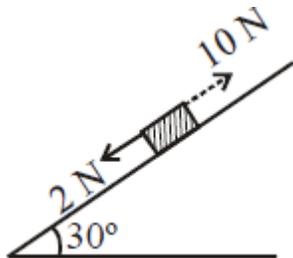
i.e, $v_{gG} = v_{ge} + v_{eG}$

$$\frac{d}{t} = \frac{d}{t_1} + \frac{d}{t_2}$$

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

$$t = \frac{t_1 t_2}{(t_1 + t_2)}$$

14. A passenger train of length 60m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. Find the ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction, and (ii) in the opposite directions. 3
15. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. Find the coefficient of static friction between the block and the plane. [Take $g = 10 \text{ m/s}^2$] 3



$$2 + mg \sin 30 = \mu mg \cos 30^\circ$$

$$10 = mg \sin 30 + \mu mg \cos 30^\circ$$

$$= 2\mu mg \cos 30 - 2$$

$$6 = \mu mg \cos 30$$

$$4 = mg \sin 30$$

$$\frac{3}{2} = \mu \times \sqrt{3}$$

$$\mu = \frac{\sqrt{3}}{2}$$

16. Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take g constant with a value 10 m/s^2 . Find the work done by the (i) gravitational force and the (ii) resistive force of air. 3

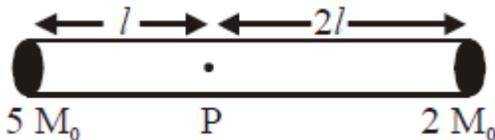
Sol. $w_g + w_a = K_f - K_i$

$$mgh + w_a = \frac{1}{2}mv^2 - 0$$

$$10^{-3} \times 10 \times 10^3 + w_a = \frac{1}{2} \times 10^{-3} \times (50)^2$$

$w_a = -8.75 \text{ J}$ i.e. work done due to air resistance and work done due to gravity = 10 J

17. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, find its instantaneous angular acceleration? 3



Ans:



Applying torque equation about point P.

$$2M_0 (2l) - 5M_0 gl = I\alpha$$

$$I = 2M_0 (2l)^2 + 5M_0 l^2 = 13M_0 l^2$$

$$\therefore \alpha = -\frac{M_0 gl}{13M_0 l^2} \Rightarrow \alpha = -\frac{g}{13l}$$

$$\therefore \alpha = \frac{g}{13l} \text{ anticlockwise}$$

18. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then find the value of d ? 3

Sol. Above earth surface

$$g' = g \left(1 - \frac{2h}{R_e} \right)$$

$$\Delta g' = g \frac{2h}{R_e} \dots(1)$$

From (1) & (2)

$$d = 2h$$

$$d = 2 \times 1 \text{ km}$$

Below earth surface

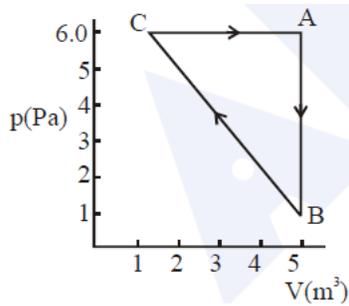
$$g' = g \left(1 - \frac{d}{R_e} \right)$$

$$\Delta g = g \frac{d}{R_e} \dots(2)$$



19. For the given cyclic process CAB as shown for a gas, Find the work done.

3



Since P–V indicator diagram is given, so work done by gas is area under the cyclic diagram.

$$\begin{aligned} \therefore \Delta W &= \text{Work done by gas} = \frac{1}{2} \times 4 \times 5 \text{ J} \\ &= 10 \text{ J} \end{aligned}$$

20. Two satellites, A and B, have masses m and $2m$ respectively. A is in a circular orbit of radius R , and B is in a circular orbit of radius $2R$ around the earth. Find the ratio of their kinetic energies, T_A/T_B .

3

Sol. Orbital velocity $V = \sqrt{\frac{GM_e}{r}}$

$$T_A = \frac{1}{2} m_A V_A^2$$

$$T_B = \frac{1}{2} m_B V_B^2$$

$$\Rightarrow \frac{T_A}{T_B} = \frac{m \times \frac{GM}{R}}{2m \times \frac{GM}{2R}}$$

$$\Rightarrow \frac{T_A}{T_B} = 1$$

21. What is capillarity? Derive an expression for the height to which the liquid rises in a capillary tube of radius r ?

3

22. State Newton's law of cooling. Deduce the relations : $\log_e(T-T_0) = -kt+c$ and where the symbols have their usual meanings. Represent Newton's law of cooling graphically by using each of the above equations.

3

23. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450 t - 9x)$ where distance and time are measured in SI units. Find the tension in the string?

3

Sol. $y = 0.03 \sin(450 t - 9x)$

$$v = \frac{\omega}{k} = \frac{450}{9} = 50 \text{ m/s}$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{T}{\mu} = 2500$$

$$\begin{aligned} \Rightarrow T &= 2500 \times 5 \times 10^{-3} \\ &= 12.5 \text{ N} \end{aligned}$$

OR

A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, find its periodic time ?

$$v = \omega \sqrt{A^2 - x^2} \quad \text{---(1)}$$

$$a = -\omega^2 x \quad \text{---(2)}$$

$$|v| = |a| \quad \text{---(3)}$$

$$\omega \sqrt{A^2 - x^2} = \omega^2 x$$

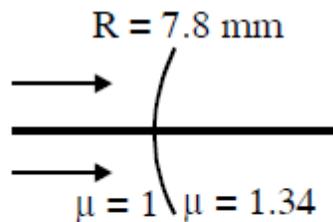
$$A^2 - x^2 = \omega^2 x^2$$

$$5^2 - 4^2 = \omega^2 (4^2)$$

$$\Rightarrow 3 = \omega \times 4$$

$$T = 2\pi/\omega$$

24. The eye can be regarded as a single refracting surface . The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus. 3



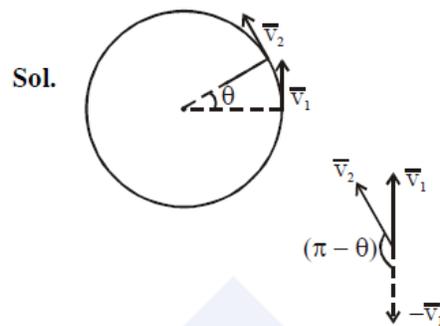
$$\frac{1.34}{V} - \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

$$\therefore V = 30.7 \text{ mm}$$

25. (a) Draw a labelled ray diagram showing the formation of image by a compound microscope in normal adjustment. Derive the expression for its magnifying power.
(b) You are given two converging lenses of focal lengths 1.25 cm and 5 cm to design a compound microscope. If it is desired to have a magnification of 30, find out the separation between the objective and the eyepiece. 5

$$\begin{aligned} M &= m_o \times m_e \\ &= \frac{L}{f_o} \left(1 + \frac{D}{f_e} \right) \\ \therefore 30 &= \frac{L}{1.25} \left(1 + \frac{25}{5} \right) \\ 30 \times 1.25 &= L \times 6 \\ L &= 5 \times 1.25 \\ &= 6.25 \text{ cm} \end{aligned}$$

26. (a) A particle is moving along a circular path with a constant speed of 10 ms^{-1} . What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? 5

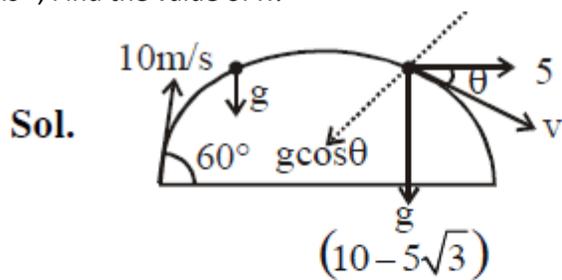


$$\begin{aligned}
 |\Delta \vec{v}| &= \sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos(\pi - \theta)} \\
 &= 2v \sin \frac{\theta}{2} \quad \text{since } [|\vec{v}_1| = |\vec{v}_2|] \\
 &= (2 \times 10) \times \sin(30^\circ) \\
 &= 10 \text{ m/s}
 \end{aligned}$$

- (b) Derive an equation of path followed by a projectile when projected with a speed 'u' making an angle ' θ ' with horizontal?

OR

- A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1 \text{ s}$ is R . Neglecting air resistance and taking acceleration due to gravity $g = 10 \text{ ms}^{-2}$, Find the value of R ? 5



$$v_x = 10 \cos 60^\circ = 5 \text{ m/s}$$

$$v_y = 10 \cos 30^\circ = 5\sqrt{3} \text{ m/s}$$

velocity after $t = 1 \text{ sec}$.

$$v_x = 5 \text{ m/s}$$

$$v_y = \left| (5\sqrt{3} - 10) \right| \text{ m/s} = 10 - 5\sqrt{3}$$

$$a_n = \frac{v^2}{R} \Rightarrow R = \frac{v_x^2 + v_y^2}{a_n} = \frac{25 + 100 + 75 - 100\sqrt{3}}{10 \cos \theta}$$

$$\tan \theta = \frac{10 - 5\sqrt{3}}{5} = 2 - \sqrt{3} \Rightarrow \theta = 15^\circ$$

$$R = \frac{100(2 - \sqrt{3})}{10 \cos 15} = 2.8 \text{m}$$

27. What is Carnot engine? Explain the construction and various operations for Carnot's heat engine working between two temperatures. Hence derive from it the efficiency of the engine. On what factors does it depend?