Session: 2018-19

Subject: Mathematics

Class – XII

Time: 3 hrs. M.M: 100

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into four sections A, B, C and D. Section A contains 4 questions of one mark each, Section B comprises of 8 questions of two marks each. Section C comprises 11 questions of four marks each and Section D comprises of 6 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 1 question of one mark each, 3 questions of two marks each, 3 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

(vi) Do not write anything on the question paper other than your Roll No.

SECTION – A

1. If matrix A = diag(d_1, d_2, d_3), then A^{-1} = diag(d_1^{-1}, d_2^{-1}, d_3^{-1}). State true or false with explanation.

2. Differentiate w.r.t. x: $\dfrac{8^x}{x^8}$

3. Write the sum of the order and degree of the differential equation:
$$\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$$

4. Write the vector parallel to the line $\dfrac{x+2}{3} = \dfrac{y-1}{2} = z$

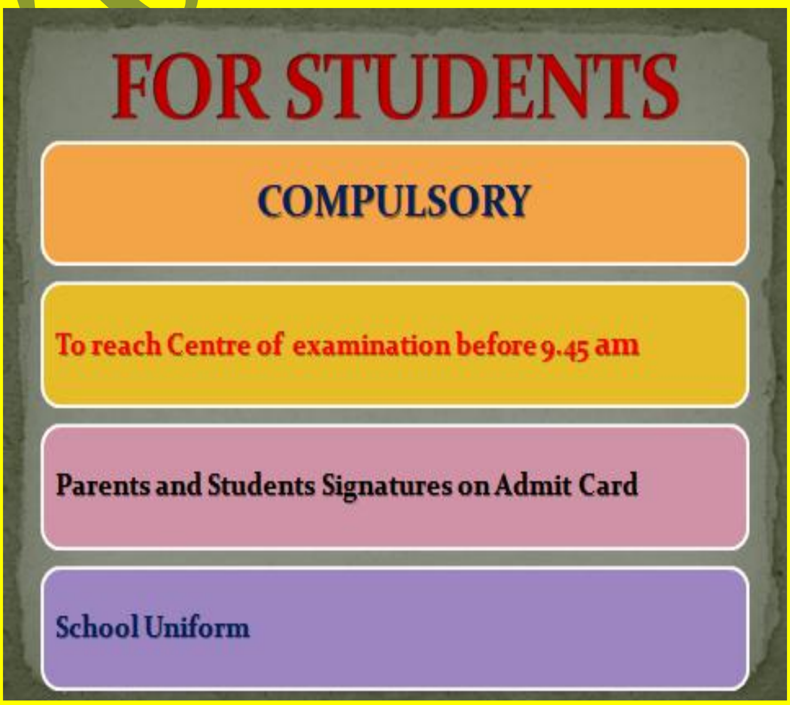
OR

If a line makes angles 90°, 135°, 45° with the x, y and z axis respectively, find its direction cosines.

SECTION – B

5. Prove that $\cos\left(2\sin^{-1}\dfrac{1}{5\sqrt{2}}\right) = \sin\left(4\cos^{-1}\dfrac{3}{\sqrt{10}}\right)$.

6. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute.



7. Find: $\displaystyle\int \frac{(x-4)e^x}{(x-2)^3}dx$

8. Find: $\displaystyle\int \frac{\sin x}{\sin(x+a)}dx$

OR $\displaystyle\int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx$

9. Verify that the function y = a cox – b sinx, where a, b ∈ R is a solution of the differential equation $\dfrac{d^2y}{dx^2} + y = 0$.

10. Find α and β if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\alpha\hat{j}+\beta\hat{k})=\vec{0}$.

OR

If $\hat{a}$ and $\hat{b}$ are two unit vectors such that their difference is also a unit vector, find angle between them.

11. A couple has two children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older is a boy.

12. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining third six in the sixth throw of the die.

OR

If A, B are independent events, then prove that A and B′ are also independent events.

SECTION – C

13. Show that the function f(x) = |x| - |x + 1| is not differentiable at x = -1.

14. If $\Delta=\begin{vmatrix}1 & a & a^2\\ a & a^2 & 1\\ a^2 & 1 & a\end{vmatrix}=-4$ then find the value of $\begin{vmatrix}a^3-1 & 0 & a-a^4\\ 0 & a-a^4 & a^3-1\\ a-a^4 & a^3-1 & 0\end{vmatrix}$.

15. Evaluate: $\int_0^4 |x^2-4|\,dx$

16. Solve for x: $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{4}, x>0$

17. On the set {0, 1, 2, 3, 4, 5, 6}, a binary operation * is defined as:

$$a*b=\begin{cases}a+b, & if\ a+b<7\\ a+b-7, & if\ a+b\geq 7\end{cases}$$

Write the operation table of the operation * and prove that zero is the identity for this operation and each element a ≠ 0 of the set is invertible with 7 – a being the inverse of a.

18. Differentiate w.r.t. x: $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$.

OR

If $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, y = a sin t, then find $\frac{d^2y}{dt^2}$, $\frac{d^2y}{dx^2}$.

19. Integrate: $\int \frac{dx}{(1+x^2)(\tan^{-1}x+1)^2(2\tan^{-1}x-3)}$ **OR** $\int \frac{xdx}{(x^2-3)\sqrt{1+x^2}}$

20. If the straight line x cos α + y sin α = p touches the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

OR

Find the equations of the normal to the curve $y = 4x^3 - 3x + 5$ which are perpendicular to the line 9x – y + 5 = 0.

21. Solve the differential equation: $\frac{dy}{dx} = \frac{x(2y-x)}{x(2y+x)}$, if y(1) = 1.

OR

Solve: x cosy dy = $(x e^x \log x + e^x)$ dx.

22. The magnitude of the vector product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

23. Find the equation of plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}-2\hat{j}+3\hat{k})-4=0$ and $\vec{r}.(-2\hat{i}+\hat{j}+\hat{k})+5=0$ and whose intercept on x-axis is equal to that of on y-axis.

Use only Blue ball point/ Gel pen

Students must maintain discipline in Examination Hall

Students may have to sign the Undertaking in the Examination Hall

Go through all Instructions properly on Admit card

Read the Instructions properly given on the Question paper

SECTION – D

24. For a given curved surface area of a right circular cone, when the volume is maximum, prove that the semi vertical angle $\theta = \sin^{-1}\frac{1}{\sqrt{3}}$.

25. Find the coordinates of the point where the line through the points (3, -4, -5) and (2, -3, 1) crosses the plane determined by the points (1, 2, 3), (4, 2, -3) and (0, 4, 3).

OR

Find the distance of the point (-2, 3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane 4x + 12y – 3z + 1 = 0.

26. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

x + 3z = 9, -x + 2y – 2z = 4 and 2x – 3y + 4z = -3.

OR

If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, find the inverse of A using elementary transformations and hence solve the following matrix equation XA = [1 0 1].

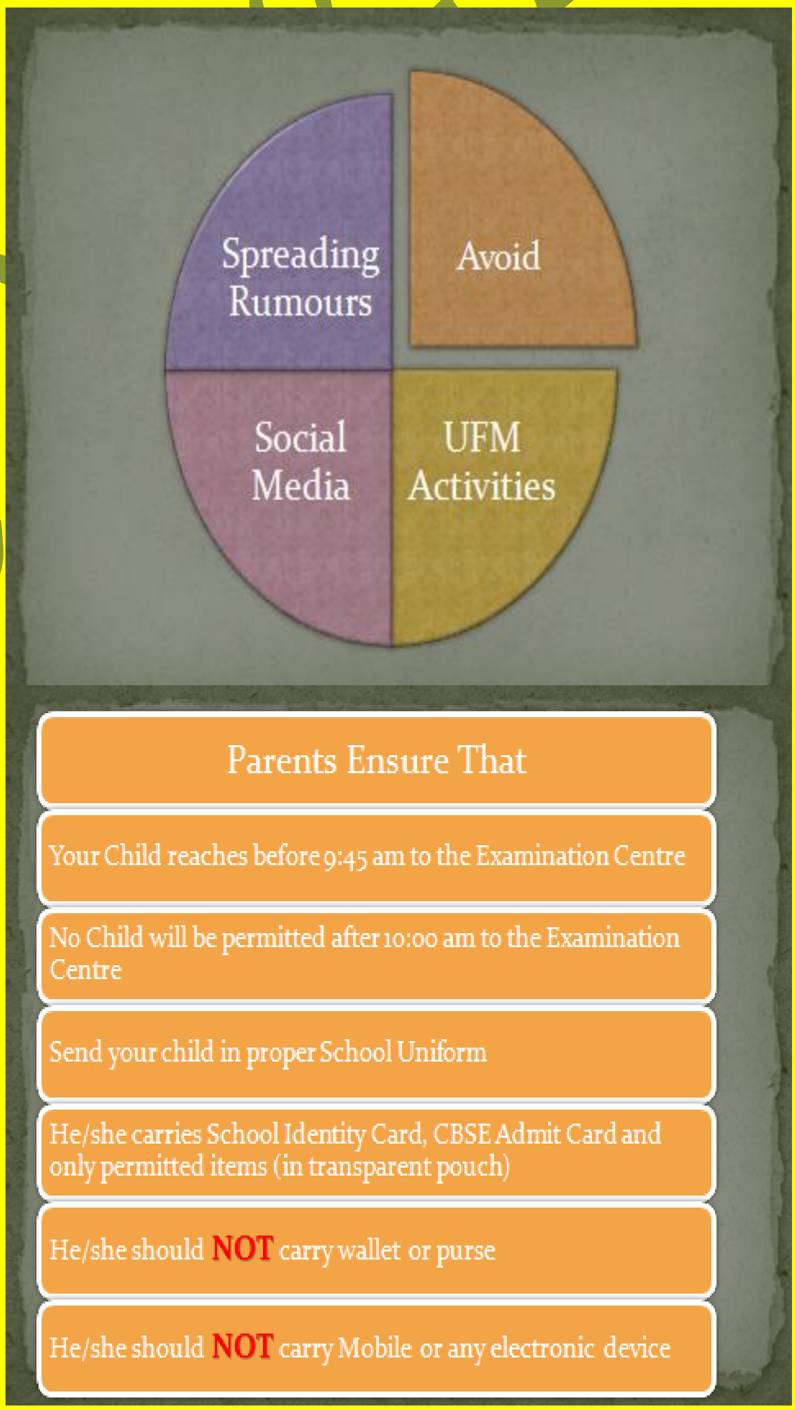
27. Find the area bounded by the curves y = √x, 2y + 3 = x and x-axis.

OR

Evaluate $\int_1^3 \left(e^{2+3x} + 2x^2 - 5\right)dx$ as a limit of sum.

28. A small scale factory makes two types of dolls. One doll of type I takes 1.5 hours of electronic machine and 3 hours of hand operated machine; one doll of type II takes 3 hours of electronic machine and 1 hour of hand operated machine. In a day, the factory has the availability of atmost 42 hours of electronic machines and 24 hours of hand operated machines. If the profit on one doll of type I is Rs 20 and on one doll of type II is Rs 30, find the number of dolls of each type that the factory should manufacture to earn maximum profit. Make it as LPP and solve graphically.

29. A bag contains (2n + 1) coins. It is known that n of these coins have a head on both sides where as the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n.



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