

SAMPLE PAPER

CLASS XII

MATHEMATICS

M.M.100

TIME: 3HOURS

SECTION A [1x 4=4]

1. If $[1 \ 2 \ 3] \begin{bmatrix} 4 \\ x \\ 2x \end{bmatrix} = [0]$, find the value of x .

2. Show that the function $f(x) = x^3 - 3x^2 + 4x$, x is a real number is strictly increasing.

3. Find the value of $\int_0^2 [x] dx$.

4. Show that $|\hat{a} \cdot \hat{b}| = 2 \sin^2 \frac{\theta}{2}$

OR

Find the value of 'p' if the vectors are $\vec{a} = \hat{i} + 2\hat{j} - p\hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} - \hat{k}$ are orthogonal.

SECTION B [2x8=16]

5. Consider the $f: \mathbb{R}_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y - 4}$, where \mathbb{R}_+ is the set of all non-negative real numbers.

6. Find the value of X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$

7. Form the differential equation of the family of hyperbolas having foci on x-axis and center at origin.

OR

Show that $y'' + 4y = 0$ if $y = A \cos 2x + B \sin 2x$

8. Find the area of the rhombus whose diagonals are $\vec{a} = 2\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} - 3\hat{j} - \hat{k}$.

9. Evaluate $\int_0^{\pi/2} \cos^2 x \, dx$

10. A pair of dice is thrown 7 times. What is the probability of getting a sum 7 atmost 7 times.

11. Evaluate $\int \frac{xe^x}{(1+x)^2} dx$

12. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

OR

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

SECTION C [4x11=44]

13 Prove that $\begin{vmatrix} a^2 & bc & ac + b^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

14. Find the particular solution of the differential equation $(1+e^2x)dy + (1+y^2)dx=0$ given that $y = 1$ when $x = 0$.

OR

Solve: $ye^y dx = (xe^y + y^2)dy$.

15. Evaluate $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$

16. Find the equation of the line passing through $(1, 2, -4)$ and perpendicular to $\vec{r} = \hat{i} + \hat{j} - \hat{k} + s(3\hat{i} - \hat{j})$ and $\vec{r} = 4\hat{i} - \hat{k} + t(2\hat{i} + \hat{k})$.

17. Show that the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$, given by

$R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation.

18. Solve: $\tan^{-1} \left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right]$, $|x| < 1$, $y > 0$ and $xy < 1$.

OR

Prove that $\cot^{-1} \left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right] = \frac{x}{2}$

19. Evaluate $\int \frac{x^4 dx}{(x-1)(x^2+1)}$

OR

Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

20. Find k, if $f(x) = \begin{cases} \frac{\log(1+x) - \log(1-x)}{x}, & \text{if } x \neq 0, \\ k, & \text{if } x = 0 \end{cases}$, is continuous at $x = 0$.

21. Using differentials find the approximate value of $\sqrt{49.3}$ upto 3 places of decimal.

22. Find the intervals in which the function f is given by $f(x) = \frac{4\sin x - 2x - \cos x}{2 + \cos x}$, $x \neq 0$, is increasing and decreasing.

23. Find two vectors of unit length which make angle 45° with $(1,0,0)$ and are at right angle to $(0,0,1)$.

SECTION D [6x6=36]

24. If $A = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix}$, find the A^{-1} and solve $x - 2y = -3$, $2x - y - z = -3$ and $-2y + z = -1$

25. Find the area of the region bounded by $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$, which lies on the 1st quadrant.

OR

Evaluate $\int_1^4 (x^3 + x^2 - 1) dx$ as limit of sum.

26. Find the image of the point $(1,2,3)$ to the line $\frac{x-2}{1} = \frac{2-y}{3} = \frac{2z-1}{-1}$

27. An aeroplane can carry 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economical class ticket. The airlines reserves at least 20 seats for executive class. However at least 4 times as many passengers prefer to travel by economic class than executive class. Determine how many tickets of each class must be sold in order to maximize the profit for the airline. What is the maximum profit?

28. In a bag there are 4 gold coins, two coins are drawn and found to be gold. What is the probability that other coins are also gold?

OR

A card from a pack of 52 cards is lost . From the remaining cards of the pack , two cards are drawn and are found to be both diamonds Find the probability of the lost cards being a diamond..

29. Show that the semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find its maximum volume.

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