PLAY WITH MATH

BHAGALPUR

Class 12 - Mathematics

TEST NO. - 01

Maximum Marks: 81 **Time Allowed: 3 hours**

General Instructions:

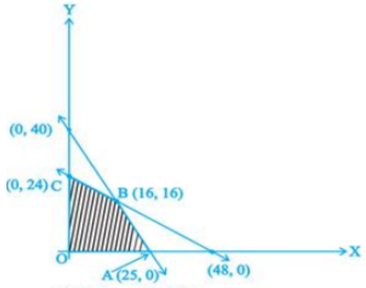
All questions are compulsory.

Section A contains 20 questions each of 1 mark. Section B contains 6 questions each of 2 marks. Section C contains 6 questions each of 4 marks. Section D contains 4 questions each of 6 marks.

Section A

1. Write fog , f : R $\rightarrow$ R and g:R$\rightarrow$ R are given by f(x) = |x| and g(x) = |5x - 2|.
2. Let R is the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b): 2 divides (a - b)}. Write the equivalence class [0].
3. If f is an invertible function, defined as f(x) = $\frac{3x-4}{5}$, then write $f^{-1}(x)$.
4. Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.
5. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, then write the value of x.
6. If A is a non-singular matrix of order 3 and | adj A| = |A| then what is the value of k?
7. Find $\frac{dy}{dx}, y=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$
8. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$
9. Find the value of the constant k so that the function f defined below is continuous at $x=0,$ Where $f(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2}, x\neq 0\\ k,\ x=0\end{cases}$
10. $\int\frac{\sec^2 x}{\cos ec^2 x}dx$
11. Evaluate $\int_0^3\frac{dx}{9+x^2}$.
12. $\int\cos^3 x.\, e^{\log\sin x}dx$
13. Write the sum of the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3+x^4=0$.
14. Find $|\vec{x}|$. if for a unit Vector $\hat{a}$ $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=15.$

15. If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ then find the projection of $\vec{a}$ on $\vec{b}$.
16. Find the vector equation of the plane with intercepts 3, -4 and 2 on X,Y and Z-axes, respectively.
17. Find the angle between the line $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$.
18. Determine the maximum value of Z = 4x + 3y if the feasible region for an LPP is shown in Figure.



19. Evaluate $P(A \cap B)$ if 2P(A) = P (B) = $\frac{5}{13}$ and $P(A|B) = \frac{2}{5}$
20. Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find: P(neither A nor B)

Section B

21. Check the injectivity and surjectivity of the following function:

    f : N $\rightarrow$ N given by f(x) = x^2
22. Without expanding, prove that $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$
23. Verify Rolle's theorem for f(x) = x^2 + 2x - 8, $x \in [-4, 2]$.
24. Integrate the function $\sqrt{x^2 + 4x + 1}$
25. Find probability of throwing at most 2 sixes in 6 throws of a single die.
26. Solve the equation: 2$\tan^{-1}$(cosx) = $\tan^{-1}$(2cosec x)

Section C

27. Find $\frac{dy}{dx}$, If y = $(\cos x)^x + (\sin x)^{1/x}$.

OR

Find $\frac{dy}{dx}$, if y = $\sin^{-1}\left[\frac{6x-4\sqrt{1-4x^2}}{5}\right]$.
28. $\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sqrt{\sin 2x}} dx$

OR

Evaluate $\int \left(\frac{1+\sin x}{1+\cos x}\right) e^x dx.$
29. Find the particular solution of the differential equation satisfying the given condition.

$x^2dy + (xy + y^2)dx = 0$, when y(1) = 1

OR

Solve $\left[x\sin^2\left(\frac{y}{x}\right) - y\right] dx + xdy = 0;$

$y = \pi/4$, when x = 1

30. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3, \left|\vec{b}\right| = 4, |\vec{c}| = 5$and each one of them being $\perp$ to the sum of the other two, find $\left|\vec{a} + \vec{b} + \vec{c}\right|$

31. Maximise and Minimise Z = 3x– 4y. subject to

$x - 2y \leqslant 0$

$-3x + y \leqslant 0$

$x - y \leqslant 6$

$x, y \geqslant 0$

32. In a school, it is known that 30% students have 100% attendance and 70% students are irregular. Previous years results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

Section D

33. Using elementary transformation, find the inverse of the matrices

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

OR

Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

34. Find the area of the region in the first quadrant enclosed by the x-axis, the line y = x, and the circle $x^2 + y^2 = 32$.

35. Find the coordinate where the line thorough (3, -4, -5) and (2, -3, 1) crosses the plane 2x + y + z = 7.

36. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

OR

A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, then find the dimensions of the rectangle that will produce the largest area of the window.