RAVI MATHS TUITION CENTER PH - 8056206308

MATRICES 2 M TEST

12th Standard CBSE

Maths

Reg.No. : | | | | | | |

Time : 00:30:00 Hrs

Total Marks : 30

1) Find the value of x, y, z if $\begin{bmatrix} 2x+y & x-y \\ x-z & x+y+z \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 2 & 8 \end{bmatrix}$ **2**

Answer : We have, $\begin{bmatrix} 2x+y & x-y \\ x-z & x+y+z \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 2 & 8 \end{bmatrix}$

$\Rightarrow$

2x + y = 10, x - y = - 1x - z = 2 and x + y + z = 8.$\therefore$

2(y - 1) + y = 10 $\Rightarrow$

2y + y + 2 = 10$\Rightarrow$

3y = 12 $\Rightarrow$

y = 4.$\therefore$

x = 33 - z, $\Rightarrow$

z = 1.$\therefore$

x = 3, y = 4, z = 1

2) If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 1 \end{bmatrix} B = \begin{bmatrix} 5 & 2 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$, then find the matrix X for which A + B - X = 0. **2**

Answer : We have A + B - X = 0By adding X on both the sides,A + B - X + X = 0 + X$\Rightarrow$

A + B = X$\Rightarrow$ $X = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow \quad X = \begin{bmatrix} 6 & 6 \\ 2 & 2 \\ 3 & 2 \end{bmatrix}$

3) Solve the matrix equation $\begin{bmatrix}x^2\\y^2\end{bmatrix} - 3\begin{bmatrix}x\\2y\end{bmatrix} = \begin{bmatrix}-2\\-9\end{bmatrix}$ **2**

Answer : We have, $\begin{bmatrix}x^2\\y^2\end{bmatrix} - 3\begin{bmatrix}x\\2y\end{bmatrix} = \begin{bmatrix}-2\\-9\end{bmatrix}$

$\Rightarrow$

$x^2 - 3x = -2$ and
$y^2 - 6y = -9 \Rightarrow$

$x^2 - 3x + 2 = 0$ and
$y^2 - 6y + 9 = 0 \Rightarrow$

$x^2 - 2x - x + 2 = 0$
and $y^2 - 3y - 3y + 9 = 0 \Rightarrow$

$x(x - 2) - 1(x - 2) = 0$ and $y(y - 3)$
$- 3(y - 3) = 0 \Rightarrow$

$(x - 2)(x - 1) = 0$ and $(y - 3)(y - 3)$
$= 0 \therefore$

$x = 1, 2$ and $y = 3, 3$

4) Find the value of X and Y if $X + Y = \begin{bmatrix}2 & 3\\5 & 1\end{bmatrix}, X - Y = \begin{bmatrix}6 & 5\\7 & 3\end{bmatrix}$ **2**

Answer : We have, $X + Y = \begin{bmatrix}2 & 3\\5 & 1\end{bmatrix}, X - Y = \begin{bmatrix}6 & 5\\7 & 3\end{bmatrix}$

$$(X + Y) + (X - Y) = \begin{bmatrix}2 & 3\\5 & 1\end{bmatrix} + \begin{bmatrix}6 & 5\\7 & 3\end{bmatrix}$$

$$2X = \begin{bmatrix}8 & 8\\12 & 4\end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix}4 & 4\\6 & 2\end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix}4 & 4\\6 & 2\end{bmatrix}$$

and $Y = \begin{bmatrix}2 & 3\\5 & 1\end{bmatrix} - \begin{bmatrix}4 & 4\\6 & 2\end{bmatrix}$

$$= \begin{bmatrix}-2 & -1\\-1 & -1\end{bmatrix}$$

5) If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$, show that $AB \neq BA$. **2**

Answer : We have, $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

and $B = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} -2+0 & 8+2 \\ -3+0 & 12+8 \end{bmatrix}$

$\begin{bmatrix} -2 & 10 \\ -3 & 20 \end{bmatrix}$

and $BA = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 10 & 15 \\ 6 & 8 \end{bmatrix}$

$\therefore$

$AB \neq BA$

6) If $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$, find AB and BA. **2**

Answer : We have, $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

and $B = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

$\Rightarrow$

AB = [-2 + 6 + 3] = [7]and $BA = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

$\Rightarrow \quad BA = \begin{bmatrix} -2 & -4 & -6 \\ 3 & 6 & 9 \\ 1 & 2 & 3 \end{bmatrix}$

7) Find the value of x and y in each if AB exist (i) $A_{3\times x}, B_{4\times y}$ and $AB_{3\times 3}$ (ii) $A_{x\times 2}, B_{y\times 4}$ and $AB_{3\times 4}$ **2**

Answer : We have, $A_{3\times x}$

and $B_{4\times y}$

= $AB_{3\times 3}$

(i) $(A_{3\times x},)\,(B_{4\times y}) = (AB_{3\times 3})$

$\therefore$

x = 4 and y = 3(ii) $(A_{x\times 2})\,(B_{y\times 4}) = (AB_{3\times 4})$

$\therefore$

y = 2, x = 3

8) If is $A = \begin{bmatrix} 0 & b & -2 \\ 3 & 1 & 3 \\ 2a & 3 & -1 \end{bmatrix}$ skew symmetric matrix, find the values of a and b. **2**

Answer : If A is

symmetric matrix then$A = A'$

$$\Rightarrow \quad \begin{bmatrix} 0 & b & -2 \\ 3 & 1 & 3 \\ 2a & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2a \\ b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix}$$

$\therefore$

By equality of matrices, b = 3 and a = -

1

9) If $A = \begin{bmatrix} 0 & x & -4 \\ -2 & 0 & -1 \\ y & -1 & 0 \end{bmatrix}$ is skew symmetric matrix, find the values of x and y. **2**

Answer : $A = \begin{bmatrix} 0 & x & -4 \\ -2 & 0 & -1 \\ y & -1 & 0 \end{bmatrix}$

$$A' = \begin{bmatrix} 0 & -2 & y \\ x & 3 & -1 \\ -4 & -1 & 0 \end{bmatrix}$$

For skew symmetric$A = -A'$

$$\Rightarrow \quad \begin{bmatrix} 0 & x & -4 \\ -2 & 0 & -1 \\ y & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -2 & y \\ x & 3 & -1 \\ -4 & -1 & 0 \end{bmatrix}$$

x = 2, y = 4

10) Find $\frac{1}{2}\left(A+A'\right)$ and $\frac{1}{2}\left(A-A'\right)$. If $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$ 2

Answer : We have, $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

$$A'=\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$$

$$A+A'=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\Rightarrow \quad \frac{1}{2}A+A'=\frac{1}{2}\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=0$$

and $A-A'=\begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix}$

$$\Rightarrow \quad \frac{1}{2}A-A'=\frac{1}{2}\begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix}$$

$$\Rightarrow \quad A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$$

11) If $\begin{bmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{bmatrix}$, find the value of θ satisfying the equation A + A^T = I_2 , where $0 \leq 0 \leq \frac{\pi}{2}$. 2

Answer : We have, $A=\begin{bmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{bmatrix}$

$$A^T=\begin{bmatrix}cos\theta & sin\theta\\ -sin\theta & cos\theta\end{bmatrix}$$

$$\Rightarrow \quad A+A^T=\begin{bmatrix}2cos\theta & 0\\ 0 & cos\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \quad 2cos\theta=1$$

$$\Rightarrow \quad cos\theta=\frac{1}{2}\Rightarrow\theta=\cos^{-1}\left(\frac{1}{2}\right)$$

$$\because \quad \theta=\frac{\pi}{3}$$

12) Prove that the diagonal elements of a skew symmetric matrix are all zero. **2**

Answer : Let

A be a skew-symmetric matrix. Then by definition $A' = -A$

$\Rightarrow$

the (i, j)th element of A'

= the (i, j)th element of (- A)$\Rightarrow$

the (j, i)th element of A = - the (i, j)th element of
AFor the diagonal elements i = j $\Rightarrow$

the (i, j)the element of A = - the (i, j)th element of
A.$\Rightarrow$

the (i, j)th element of A = 0Hence the diagonal
elements are all zero.

13) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, then $n \epsilon N$. **2**

Answer : We shall prove the result by using principle of mathematical induction.Let $P(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$

Now, $P(1): A^1 = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

The result is true for n = 1.Let the result be true for n = k.So, $A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$

Now, we prove that P(k + 1) is true.Now,

$A^{k+1} = A. A^k = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$

$= \begin{bmatrix} 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \end{bmatrix}$

$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$

$= A^{k+1}$Hence, it is true n = k + 1.Hence,
by principle of mathematical induction P(n) is true for all $n \epsilon N$

14) Prove the following by the principle of mathematical induction :if $A=\begin{bmatrix}3 & -4\\1 & -1\end{bmatrix}$, then $A^n=\begin{bmatrix}1+2n & -4n\\n & 1-2n\end{bmatrix}$ for every positive integer n. **2**

Answer : We shall prove the result by mathematical induction on n.Step 1 : When n = 1, by the definition or integral powers of a matrix, we have $A^1=\begin{bmatrix}1+2(1) & -4n\\n & 1-2(1)\end{bmatrix}=\begin{bmatrix}3 & -4\\1 & -1\end{bmatrix}$

So, the result is true for n = 1.Step 2 : Let the result be true for n = m. Then, $A^m=\begin{bmatrix}1+2m & -4m\\m & 1-2m\end{bmatrix}$

Now, we will show that the result is true for n = m + 1,
i.e., $A^{m+1}=\begin{bmatrix}1+2(m+1) & -4(m+1)\\(m+1) & 1-2(m+1)\end{bmatrix}$

By the definition of integral powers of a square matrix, we have $A^{m+1}=A^m.A$

$$\Rightarrow \quad A^{m+1}=\begin{bmatrix}1+2m & -4m\\m & 1-2m\end{bmatrix}\begin{bmatrix}3 & -4\\1 & -1\end{bmatrix}$$

[by supposition (i)] $\Rightarrow \quad A^{m+1}=\begin{bmatrix}3+6m-4m & -4-8m+4m\\3m+1-2m & -4m-4+2m\end{bmatrix}$

$$=\begin{bmatrix}1+2(m+1) & -4(m+1)\\(m+1) & 1-2(m+1)\end{bmatrix}$$

This shows that the result is true for n = m + 1, whenever it is true for n = m.Hence, by the principle of mathematical induction, the result is true for any positive integer n.

15) Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and skew symmetric matrix. **2**

Answer : Let A be any square matrix. Then, $A=\frac{1}{2}\left(A+A^T\right)+\frac{1}{2}\left(A-A^T\right)$

= P + Q (say),where, $P=\frac{1}{2}\left(A+A^T\right)$

and $Q=\frac{1}{2}\left(A-A^T\right)$

Now, $P^T=\left[\frac{1}{2}\left(A+A^T\right)\right]^T$

$=\frac{1}{2}\left(A+A^T\right)^T \quad \left[\because \quad (KA)^T=K.A^T\right]$

$\Rightarrow \quad P^T=\frac{1}{2}\left[A^T+\left(A^T\right)^T\right]$

$\left[\because \quad (A+B)^T=A^T+B^T\right]$

$\Rightarrow \quad P^T=\frac{1}{2}\left(A^T+A\right)$

$\left[\because \quad \left(A^I\right)^T=A\right]$

$\Rightarrow \quad P^T=\frac{1}{2}\left(A+A^T\right)=P$

$\therefore$

P is symmetric matrix.Also, $Q^T=\frac{1}{2}\left(A-A^T\right)^T$

$=\frac{1}{2}\left(A-A^T\right)^T$

$=\frac{1}{2}\left[A^T-\left(A^T\right)^T\right]$

$=\frac{1}{2}\left[A^T-A\right]$

$\Rightarrow \quad Q^T=\frac{1}{2}\left[A-A^T\right]=-Q$

$\therefore$

Q is skew symmetric matrix.Thus, A = P + Q,
where P is a symmetric matrix and Q is a skew symmetric matrix.Hence, A is expressible as the sum of a symmetric and a skew symmetric matrix.
Uniqueness : If possible, let A = R + S, where R is symmetric and S is skew symmetric, thenA^T = $(R + S)^T$ =

$R^T + S^T \Rightarrow$

$A^T = R - S$ (∵

$R^T = R$ and $S^T = -$

S)Now, $A = R + S$ and $A^T = R - S \Rightarrow$ $R = \frac{1}{2}\left[A + A^T\right] = P$

$S = \frac{1}{2}\left[A - A^T\right] = Q$

Hence, A is uniquely expressible as the sum of a symmetric and a skew symmetric matrix.