Rise 'n' Shine Convent School - Dhamdha

Half Yearly Examination (2019-2020)

Class – XII

Subject - Mathematics

Roll No:	Date:- 21/11/2019	Time:-3 hrs	Max. Marks : 80

General Instructions :-

1. *All questions are compulsory.*
2. The *question paper consists of 36 questions divided into four Sections A ,B,C and D* ***Section-A*** *comprises of 20 questions of one mark each,* ***Section - B*** *comprises of 6 questions of 2 marks each,* ***Section – C*** *comprises of 6 questions of 4 marks each and* ***Section – D*** *comprises of 4 questions of 6 marks each*
3. *Use of calculator is not permitted.*

SECTION - A

Q1 - Q10 are multiple choice type questions. Select the correct option

Q.1 If $y = a\sin mx + b\cos mx$,then $\frac{d^2y}{dx^2}$ is equal to

(A) $-m^2y$ (B) m^2y (C) $-my$ (D) my

Q.2 Let R be a relation on set N defined by $+2y = 8$. The domain of R is

(A){2,4,8} (B) {2,4,6,8} (C) {2,4,6, } (D) {1,2,3,4}

Q.3 If the function from $f: R \to R$ be defined by $f(x) = \frac{x}{x+1}$ then $f^{-1}(2)$ is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 2 (D) -2

Q.4 If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$,then x *is equal*

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{1}{2}$

Q.5 The value of $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$ is

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{3\pi}{4}$

Q.6 The value of $\cos^{-1}(\cos 6)$ is

(A) 6 (B) $\pi - 6$ (C) $\pi + 6$ (D) $2\pi - 6$

Q.7 If A is square matrix such that $A^2 = A$,then $(I + A)^3 - 7A$ *is equal to*

(A) A (B) $I - A$ (C) I (D) 3A

Q.8 If A= $\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$, and $A^T + A = I_2$ if θ *is equal to*

(A) $n\pi\ ,\ n \in Z$ (B) $(2n+1)\frac{\pi}{2}, n \in Z$ (C) $2n\pi + \frac{\pi}{3}, n \in Z$ (D) None of these

Q.9 If A be a square order 3×3, then the value of $|adj(A)|$ if matrix of $|A| = 2$

(A) 4 (B) 8 (C) 16 (D) 32

Q.10 $\int x^2 e^{x^3} dx$

(A) $\frac{1}{3}e^{x^2} + c$ (B) $\frac{1}{3}e^{x^3} + c$ (C) $\frac{1}{3}x^2e^{x^2} + c$ (D) $\frac{1}{3}x^3e^{x^2} + c$

Q .No 11 to Q. No 15 fill in the blank

Q.11 If f be the greatest integer function defined as f(x) = [x] and g be the modulus function defined as g(x) = |x| , then the value of $gof\left(-\frac{5}{4}\right)$ is ……

Q.12 If the following function is continuous at $x = 2$

$$f(x) = \begin{cases} \frac{x^2+3x-10}{x-2}\ , & if\ x \neq 2 \\ K & ,\ if\ x = 2 \end{cases}$$ then the value of K is ……

Q.13 If tangent to the curve $y^2 - 3x + 7 = 0$ at the point $(h, k)$is parallel to line $x - y = 4$, then the value of k is ……

Q.14 The degree of the differential equation $\left(\frac{d^2x}{dx^2}\right)^3 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is ……

Q.15 The number of arbitrary constant in the particular solution of a differential equation of second order are ……

(Q16 - Q20) Direct answer the following questions

Q.16 Check whether $(a-b)$ is factor of the determinant

$$\begin{vmatrix}(a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2\end{vmatrix}$$ or not . Give reason.

Q.17 If $f(x) = \sqrt{x^2+9}$, write the value of $\lim_{x\to 4}\frac{f(x)-f(4)}{x-4}$

Q.18 Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} sin^5x$

Q.19 Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Q.20 Find the approximate change in the volume of a cube of side x metres caused by increasing the side by 3%

SECTION - B

Q.21 Find the values of a and b such that the function f defined by

$$f(x)=\begin{cases}\dfrac{x-4}{|x-4|}+a, & if\ x<4\\ a+b & if\ x=4\\ \dfrac{x-4}{|x-4|}+b & if\ x>4\end{cases}$$ is continuous

Q.22 Integrate
$\int \frac{\sec^2 x}{\sqrt{4-\tan^2 x}}dx.$

Q.23 If $e^{x+y}=x$ then prove that $\frac{dy}{dx}=\frac{1-x}{x}$

Q.24 Find the intervals in which the function f(x) = 6 – $9x - x^2$ is strictly increasing or decreasing

Q.25 Integrate
$\int \frac{1}{(x-1)(x-2)}$

Q.26 Find the area lying in the first quadrant and bounded by the circle $x^2+y^2=4$ and the line $x=0\ and\ x=2$

SECTION - C

Q.27 If $y=log\sqrt{\frac{1+\tan x}{1-\tan x}}$,prove that $\frac{dy}{dx}=\sec 2x$

OR

Differentiate $\sin^{-1}\left\{\frac{2^{x+1}.}{1+4^x}\right\}\ w.r.t\ x$

Q.28 Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

Q.29 Find general solution of following differential equation

$$cos^2x.\frac{dy}{dx}+\ y=\tan x$$

Q.30 Evaluate
$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$

Q.31 Using Integration find area of the circle of circle $x^2+\ y^2=16$ interior to the parabola $y^2=6x$

Q.32 Solve the system of following equations by method of matrix

$$2x-3y+5z=11\ ,\ 3x+2y-4z=\ -5\ \ ,\ x+y-2z=\ -3$$

SECTION - D

Q.33 If $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ and $p \neq a, q \neq b, r \neq c$ then find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$

OR

If a, b, c are positive and unequal, showthat the value of determinant $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

Q.34 Show that height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

Q.35 Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by $f(x) = x^3 - 1$, then prove that f^{-1} exist and find f^{-1} Also find value of $f^{-1}(26)$ and $f^{-1}(-9)$

Q.36 Using method of integration find the area of the triangle ABC, the coordinates whose vertices are A(2 , 0) , B (4 , 5) and C (6 , 3)