Class – XII

MATHEMATICS- 041

SAMPLE QUESTION PAPER (4) 2019-20

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

(i) All the questions are compulsory.
(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION A

Q1 - Q10 are multiple choice type questions. Select the correct option.

1. If $A = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$, then $A + A^T = I$, then the value of α is

$a)\frac{3\pi}{2}$ $b)\ \pi$ $c)\ \frac{\pi}{6}$ $d)\frac{\pi}{3}$.

2. If A and B are square matrices of the same order, then the value of $(A+B)(A-B)$ is equal to

$a)A^2 - B^2$ $b)\ A^2 - BA - AB + B^2$ $c)\ A^2 - B^2 + BA - AB$ $d)A^2 - BA + B^2 + AB$.

3. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then the range of $|\lambda\vec{a}|$is:

(a)$[0,8]$ (b) $[-12,8]$ (c) $[0,12]$ (d) $[8,12]$.

4. Let A and B two given events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$, then $P(A'/B')$ is

(a)$\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$.

5. The maximum value of $4x+5y$ subject to the constraints $x+y \le 20$, $x+2y \le 35$, $x-3y \le 12$ is

(a) 84 (b) 95 (c)100 (d)96

6. If $tan^{-1}3 + tan^{-1}x = tan^{-1}8$, then the value of x is

$a)\frac{1}{\sqrt{3}}$ $b)\frac{1}{5}$ $c)5$ $d)\sqrt{3}$.

7. Three balls are drawn from a bag containing 2 red and 5 black balls, if the random variable X represents the number of red balls drawn, then X can take the values

(a) $0,1,2$ (b) $0,1,2,3$ (c) 0 (d) $1,2$.

8. The value of $\int \frac{1}{\sin^2 x \cos^2 x} dx$ is:

(a) $\sin^2 x - \cos^2 x + C$ (b) -1 (c) $tanx + cotx + C$ (d) $tanx - cotx + C$.

9. Distance of plane $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) + 2 = 0$, from the origin is

(a) 2 units (b) 14 units (c) $\frac{2}{7}$ units (d) None of these.

10. A line makes angle α, β, γ with x-axis, y-axis and z-axis respectively, then $cos2\alpha + cos2\beta + cos2\gamma$ is equal to:

(a) 2 (b) 1 (c) -2 (d) -1

(Q11 - Q15) Fill in the blanks

11. If $f = \{(5,2),(6,3)\}$, $g = \{(2,5),(3,6)\}$, then $f\ o\ g =$_____.

12. The function $f(x) = f(x) = \begin{cases} x+5, & if\ x \le 1 \\ x-5, & if\ \ x > 1 \end{cases}$ is continuous at all points except at $x =$ ___.

13. If $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & y & z \end{bmatrix}$ is a skew symmetric matrix, then the values of $x,\ y\ and\ z$ are_____.

14. The side of a square is increasing at the rate of 0.2cm/sec. Then the rate of increase of perimeter of the square is____.

OR

For the curve $y = (2x+1)^3$, then the rate of change of slope at $x = 1$ is____.

15. If $\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$ are two equal vectors, then the value of $x + y + z$ is_______.

OR

The angle between the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$ is____.

(Q16 - Q20) Answer the following questions

16. If *A* and *B* are square matrices of order 3 such that $|A| = 2$ and $|B| = 3$, then find the value of $|3AB|$

17. Evaluate: $\int_0^{\frac{\pi}{4}} tan\, x\, dx$.

18. Evaluate: $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx$

OR

Evaluate: $\int \frac{1}{x^2+4x+8} dx$.

19. Evaluate: $\int \frac{10x^9 + 10^x \log 10}{x^{10} + 10^x} dx$.

20. Find the product of the order and degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + y^2 = 0$.

SECTION B

21. Consider $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = 4x + 3$. Show that *f* is invertible. Find the inverse of f.

OR

Evaluate: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.

22. Find the value of *k* for which: $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & if -1 \le x < 0 \\ \frac{2x+1}{x-1} & if\ 0 \le x < 1 \end{cases}$ is continuous at $x = 0$.

23. Use differentials to approximate $\sqrt{50}$.

24. Find the value of x such that the points $A(3,2,1)$, $B(4, x, 5)$, $C(4,2,-2)$ and $D(6,5,-1)$ are coplanar.

OR

Prove that $\left(\vec{a} \times \vec{b}\right)^2 = \vec{a}^2\vec{b}^2 - (\vec{a}.\vec{b})^2$.

25. Find the equation of a line which passes through the point $(-4,\ 2, -3)$ and is parallel to the line: $\frac{-x-2}{4} = \frac{y+3}{-2} = \frac{2z-6}{3}$.

26. A die is rolled twice and the sum of the numbers on them is observed to be 7. What is the conditional probability that the number 2 has appeared at least once?

SECTION C

27. Consider $f: \mathbb{R}_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with inverse f^{-1} of f, given by $f^{-1}(x) = \sqrt{x-4}$, where $\mathbb{R}_+$ is the set of all non-negative real numbers.

28. If $x\cos(a+y) = \cos y$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence show that : $\sin a \frac{d^2y}{dx^2} + \sin 2\,(a+y)\frac{dy}{dx} = 0$.

OR

Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $x \in (-1,1), x \neq 0$.

29. Solve: $(y^3 - 2yx^2)dx + (2xy^2 - x^3)$

30. Evaluate: $\int_{-1}^{2}\left(e^{3x} + 7x - 5\right)dx$ as the limit of sum.

31. Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

OR

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation and variance of X.

32. A firm manufactures two types of products A and B and sells them at a profit of Rs.5 per unit of type A and Rs.3 per unit of type B. Each product is processed on two machines M and N. One unit of type A requires one minute of processing time on M and two minutes of processing time on N where as one unit of type B requires one minute of processing time on M and one minute on N. Machines M and N are respectively available for at most 5 hours and 6 hours a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

SECTION D

33. i) Express $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

ii) If $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, then verify that: (i) $(A^T)^T = A$ (ii) $(AB)^T = B^T A^T$

OR

Solve given system of equations by using matrix method:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2; x, y, z \neq 0.$

34. Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$, using integration.

35. Show that the semi vertical angle of a cone of maximum volume and given slant height is given by $\cos^{-1}\frac{1}{\sqrt{3}}$.

OR

Show that the height of the cylinder of greatest volume which can be inscribed in aright circular cone of height h and semi-vertical angle α is one third of that of the cone and greatest volume of the cylinder is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

36. Find the distance of the point (2, 12, 5) from the point of intersection of the lines $\vec{r} = \left(2\hat{i} - 4\hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0$.

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