

Time: 3 Hrs.

Maximum Marks: 80

General Instructions:

- (i) All the questions are compulsory.
- (ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- (iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted.

SECTION A

Q1 - Q10 are multiple choice type questions. Select the correct option.

1. If A, B are symmetric matrices of same order, then $(AB + BA)$ is a
 a) skew symmetric matrix b) identity matrix c) zero matrix d) symmetric matrix.
2. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to
 a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
3. A vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 15 is
 (a) $\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$ (b) $15\hat{i} - 30\hat{j} + 30\hat{k}$ (c) $\hat{i} - 2\hat{j} + 15\hat{k}$ (d) None of these.
4. If $P(A \cap B) = 70\%$ and $P(B) = 85\%$, then $P(A/B)$ is equal to
 (a) $\frac{14}{17}$ (b) $\frac{17}{20}$ (c) $\frac{7}{8}$ (d) $\frac{1}{8}$
5. The optimal value of the objective function is attained at the points
 (a) Given by intersection of inequations with axes only.
 (b) Given by intersection of inequations with x-axis only.
 (c) Given by corner points of the feasible region.
 (d) None of these.
6. The value of $\sin(\tan^{-1}99 + \cot^{-1}99)$ is equal to:
 a) 0 b) -1 c) $\frac{1}{2}$ d) 1.
7. Let A and B two given independent events such that $P(A) = p$ and $P(B) = q$ and $P(\text{exactly one } A, B) = \frac{2}{3}$, then the value of $3p + 3q - 6pq$ is:
 (a) 2 (b) -2 (c) 4 (d) -4.
8. The value of λ for which $\int \frac{4x^3 + \lambda 4^x}{4^x + x^4} dx = \log|4^x + x^4| + C$
 (a) 1 (b) $\log_e 4$ (c) $\log_4 e$ (d) 4.
9. Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are coplanar if
 (a) $(\vec{a}_2 - \vec{a}_1) \cdot (-\vec{b}_1 \times -\vec{b}_2) = 0$ (b) $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

$$(c) (\vec{a}_2 - \vec{a}_1) \cdot (-\vec{b}_1 \times \vec{b}_2) = 0$$

$$(d) (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times -\vec{b}_2) = 0.$$

10. Cartesian equation of a plane that passes through the intersection of two given planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is
- (a) $(A_1x + B_1y - C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$.
- (b) $(-A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$.
- (c) $(A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$.
- (d) $(A_1x - B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$.

(Q11 - Q15) Fill in the blanks

11. If f is an invertible function defined as $f(x) = \frac{3x-4}{5}$. Then $f^{-1}(x)$ is _____.

12. The value of $\lim_{x \rightarrow 0} \left[\frac{\sin^{-1} x}{x} \right] =$ _____.

13. All the diagonal elements of a skew symmetric matrix are _____.

14. The maximum values of the function $f(x) = \sin(2x + 5)$ is _____.

OR

The values of x , the function $y = x^4 - \frac{4}{3}x^3$ is increasing, _____.

15. Write the value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$.

OR

If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and angle between \vec{a} and \vec{b} is 60° , then $\vec{a} \cdot \vec{b} =$ _____.

(Q16 - Q20) Answer the following questions

16. Without expanding, prove that $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = 0$.

17. Evaluate: $\int_0^{2\pi} \cos^5 x \, dx$

18. Evaluate: $\int x e^x \, dx$.

OR

Evaluate: $\int \frac{1}{4x^2 + 25} \, dx$.

19. Integrate: $\frac{\cos 2x + 2 \sin^2 x}{\cos^2 x}$.

20. Write the integrating factor of the differential equation: $(1 + y^2) + (2xy - \cot y) \frac{dy}{dx} = 0$

SECTION B

21. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x^2 - 5$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ by $g(x) = \frac{x}{x^2+1}$. Then, find $g \circ f$.

OR

Prove that: $\sin \left[\tan^{-1} \left(\frac{1-x^2}{2x} \right) + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right] = 1, 0 < x < 1$.

22. Differentiate: $\tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$ w.r.t. x .

23. A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp post 6 metre high. Find the rate at which the length of his shadow increases.

24. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

OR

Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal.

25. The equation of a line is $\frac{2x-5}{4} = \frac{y+4}{3} = \frac{6-z}{6}$. Find the direction cosines of a line parallel to this line.

26. Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?

SECTION C

27. Let R be the relation on $\mathbb{N} \times \mathbb{N}$, defined by $(a, b)R(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$. Check whether R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$.

28. If $\frac{x}{x-y} = \log \frac{a}{x-y}$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$.

OR

If $y = (x + \sqrt{1 + x^2})^n$, then show that $(1 + x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = n^2y$.

29. Solve: $xe^{\frac{y}{x}} + y \sin\left(\frac{y}{x}\right) - x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) = 0$, $y(1) = 0$

30. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

31. A random variable X has the following probability distribution values of X :

| | | | | | | | | |
|-------------|---|---|----|----|----|----------------|-----------------|--------------------|
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P(X) | 0 | k | 2k | 2k | 3k | k ² | 2k ² | 7k ² +k |

Find each of the following: (i) k (ii) $P(X < 6)$ (iii) $P(X \geq 6)$ (iv) $P(0 < X < 5)$

OR

Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all the students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

32. Two tailors A and B earn Rs.300 and Rs.400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

SECTION D

33. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations:
 $y + 2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$.

OR

Using elementary row operations, find the inverse of the following matrix: $A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

34. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 16a^2, y^2 \leq 6ax, x, y \geq 0\}$.
35. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$.

OR

Find the equation of tangents to the curve $y = \cos(x + y)$, $x \in [-2\pi, 2\pi]$ that are parallel to the line: $x + 2y = 0$.

36. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to 2, 3, -6