IMPORTANT QUESTIONS FOR CBSE 2019-20
SUB :- MATHEMATICS (041)

Note: Please go through concepts of each and every topics and then try to solve with open mind to observe the structure of problems given for two sections.

For Section C(4 marks)

1. Let R be the relation on N X N defined by (a, b) R (c, d) if ad = bc. Prove that R is an equivalence relation.

Sol. For the relation R defined by (a, b) R (c, d) if ad=bc on N X N.

Reflexivity: Let (a, b) ∈ N X N. (a, b) R (a, b) as ab = ba (Multiplication is commutative on N) ⇒ (a, b) R (a, b) ∀ (a, b)∈ N X N.

∴ R is reflexive relation on N X N.

Symmetry: Let (a, b) R (c, d) ⇒ ad = bc ∀ a, b, c, d ∈ N

⇒ cb = da ∀ a, b, c, d ∈ N (Multiplication is commutative on N)

⇒ (c, d) R (a, b) ∀ a, b, c, d ∈ N.

∴ R is symmetric relation on N X N.

Transitivity: Let (a, b) R (c, d) ⇒ ad = bc ∀ a, b, c, d ∈ N ... (i)

& (c, d) R (e, f) ⇒ cf = de ∀ c, d, e, f ∈ N ... (ii)

Multiplying (i) and (ii) we get

adcf = bcde

Divide by cd both sides

⇒ af = be (Multiplication is commutative on N)

⇒ (a, b) R (e, f) ∀ a, b, c, d, e, f ∈ N

∴ R is transitive relation on N X N.

∵ R is reflexive, symmetric and transitive so R is equivalence relation.

OR

Show that the relation R in the set R of real numbers, defined as R = {(a, b): a, b ∈ R and a≤ b^3} is neither reflexive, nor symmetric nor transitive. (NCERT) (Delhi 2010) (AI 2010)

Sol. For the relation R = {(a, b): a, b ∈ R and a ≤ b^3} in the set R of real numbers.

Reflexivity: $\frac{1}{2} \in R, \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ but $\frac{1}{2} \nleq \frac{1}{8} \Rightarrow (\frac{1}{2}, \frac{1}{2}) \notin R \Rightarrow$ R is not reflexive relation.

Symmetry: 1≤ $(2)^3 \Rightarrow (1, 2) \in$ R but (2, 1)) ∉ R as 2 ≰ $(1)^3$ ⇒ R is not symmetric relation.

Transitivity: As 63 ≤ $(4)^3$⇒ (63, 4) ∈ R and (4, 2) ∈ R as 4 ≤ $(2)^3$ but (63, 2) ∉ R as 63 ≰ $(2)^3$

∴ R is not transitive relation on R.

OR

Let A = {1,2,3,...,9} and R be the relation in A X A defined by (a, b) R (c, d) if a + d = b + c for a, b, c, d ∈A. Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)]. (NCERT Exemplar) (Delhi 2014) (SP 15)

Sol. For the relation R defined by (a, b) R (c, d) if a + d = b + c for a, b, c, d ∈ A on the set A = {1, 2, 3,..., 9}

Reflexivity: Let (a, b) ∈ A X A. (a, b) R (a, b) as a + b = b + a (Addition is commutative on N) ⇒ (a, b) R (a, b) ∀ (a, b)∈ A X A.

∴ R is reflexive relation on A.

Symmetry: Let (a, b) R (c, d) ⇒ a + d = b + c ∀ a, b, c, d ∈ A

⇒ c + b = d + a ∀ a, b, c, d ∈A (Addition is commutative on N)

⇒ (c, d) R (a, b) ∀ a, b, c, d ∈ A.

∴ R is symmetric relation on A.

Transitivity: Let (a, b) R (c, d) ⇒ a + d = b + c ∀ a, b, c, d ∈A

& (c, d) R (e, f) ⇒ c + f = d + e ∀ c, d, e, f ∈A

⇒ a + d + c + f = b + c + d + e ∀ a, b, c, d, e, f ∈A

⇒ a + f = b + e ∀ a, b, c, d, e, f ∈ A

⇒ (a, b) R (e, f) ∀ a, b, c, d, e, f ∈ A

∴ R is transitive relation on A.

∵ R is reflexive, symmetric and transitive so R is equivalence relation.

Equivalence class [(2,5)] is {(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)}.

OR

Determine whether the relation R defined on the set R of all real numbers as R={(a,b): a,b∈ R and $a - b + \sqrt{3} \in S$, where S is the set of all irrational numbers}, is reflexive , symmetric and transitive.

OR

Let f: N→Z be defined as

f(n) = $\begin{cases} \frac{n-1}{2}, when\ n\ is\ odd \\ \frac{-n}{2}, when\ n\ is\ even \end{cases}$ for all n∈N. State whether the function is bijective. Justify your answer.

Sol. Injectivity : Let x_1 and x_2 ∈ N such that f(x_1) = f(x_2).

Case I. Let both numbers x_1 and x_2 are odd then f(x_1) = f(x_2) ⇒ $\frac{x_1-1}{2} = \frac{x_2-1}{2}$ ⇒ x_1 = x_2 ∀ x_1,x_2 ∈ N

∴ f is 1-1 function.

Case II. Let both numbers x_1 and x_2 are even then f(x_1) = f(x_2) ⇒ $\frac{-x_1}{2} = \frac{-x_2}{2}$ ⇒ x_1 = x_2 ∀ x_1,x_2 ∈ N

∴ f is 1-1 function.

So in both the cases f is 1-1.

Surjectivity : Case I. Let n is an odd natural number then y = f(n) ⇒ $y = \frac{n-1}{2}$ ⇒ 2y= n-1 ⇒ n = 2y+1

∴ Each positive integer is an image of odd natural number.

Case II. Let n is an odd natural number then y = f(n) ⇒ $y = \frac{-n}{2}$ ⇒ 2y= -n⇒ n = -2y

∴ Each negative integer is an image of even natural number.

Combining both the above cases, R_f = Co-domain of f.

∴ f is onto.

∵ f is 1-1 and onto function.

∴ f is bijective function.

OR

Let f: N →R be a function defined as $f(x) = 4x^2 + 12x + 15$, show that f : N →S, where S is range of f, is invertible find f^{-1}. (NCERT) (AIC 2013) (SP 15)

Sol. Given f: N →R, $f(x)= 4x^2 +12x+15$. Let y=f(x) ⇒ $y= 4x^2 +12x+15$ ⇒ $4x^2 +12x+15-y=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-12\pm\sqrt{(12)^2-4(4)(15-y)}}{2X4}=\frac{-12\pm4\sqrt{9-15+y}}{8}=\frac{-3\pm\sqrt{y-6}}{2}$

$x=\frac{\sqrt{y-6}-3}{2}$, as x ϵ N. Let us define a function g : S→N, where S is range of f such that $g(y)=\frac{\sqrt{y-6}-3}{2}$

$fog(y)= 4(g(y))^2+12g(y)+15= 4(\frac{\sqrt{y-6}-3}{2})^2+12(\frac{\sqrt{y-6}-3}{2})+15=y-6+9-2(\sqrt{y-6})X3+6(\sqrt{y-6}-3)+15=y$

⇒ $fog(y)=I_y$

$gof(x)=\frac{\sqrt{f(x)-6}-3}{2}=\frac{\sqrt{4x^2+12x+15-6}-3}{2}=\frac{\sqrt{4x^2+12x+9}-3}{2}=\frac{\sqrt{(2x+3)^2}-3}{2}=\frac{2x+3-3}{2}=x$

⇒ $gof(x)=I_x$

∴ fog=gof=I

∴ $f^{-1}=g \Rightarrow f^{-1}(x)=\frac{\sqrt{x-6}-3}{2}$

OR

Let A=R-{3}, B=R-{1}. Let f:A→B be defined by $f(x) = \frac{x-2}{x-3}$, for all x∈A. Then show that f is bijective. Hence find $f^{-1}(x)$. (NCERT) (DelhiC 2014)

Sol. Let $f(x_1)= f(x_2) \Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3} \Rightarrow (x_1-2)(x_2-3)=(x_1-3)(x_2-2) \Rightarrow x_1= x_2\ \forall x_1, x_2\in A$

∴ f is 1-1 function.

Let y= f(x) ⇒ $y=\frac{x-2}{x-3} \Rightarrow y(x-3)=x-2 \Rightarrow xy-3y=x-2 \Rightarrow x(y-1)=3y-2$

⇒ $x=\frac{3y-2}{y-1}$ is defined ∀ y∈B ⇒ f is into function.

∵ f is 1-1 and onto function.

∴ f is bijective function ⇒ f is invertible $f^{-1}(x)=\frac{3x-2}{x-1}$ from the above equation.

2. Find matrix X so that $X\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix}=\begin{pmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{pmatrix}$. (MT 15) (F 17) $\left(\begin{pmatrix}1 & -2\\ 2 & 0\end{pmatrix}\right)$

OR

If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$ and $=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, then find the values of a and b. (F 15) $(b=4, a=1)$

OR

On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹ 10 more. However, if there were 16 children more, every one would have got ₹ 10 less. Using matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision? (ER 16) (32, ₹ 30; to help needy people)

OR

If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$. Show that $A^2 -5A +7I =0$ and hence find (i) A^{-1} (ii) A^3 $\left((i)\begin{bmatrix}\frac{2}{7} & -\frac{1}{7}\\ \frac{1}{7} & \frac{3}{7}\end{bmatrix}\right)$

OR

Express $\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ as sum of symmetric and skew symmetric matrices.

OR

Prove that $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$ is divisible by a+b+c and find the quotient.

OR

Prove the following, using properties of determinants:

$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$ = $2(a+b+c)^3$ (Foreign10), (DelhiC 2012), (Delhi 2014)

OR

Using properties of determinants, prove the following :

$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$ = $3abc - a^3 - b^3 - c^3$ (AIC 2012) (SP2 17)

OR

Prove the following, using properties of determinants:

$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(3abc - a^3 - b^3 - c^3)$ (Foreign10)

OR

If $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$ find x. (Exemplar) $(0, -12)$

OR

Using properties of determinants, prove the following:

$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$ (B 15)

OR

Using properties of determinants, prove that

$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$ (G 15)

OR

If x, y, z are in GP, then using properties of determinants, show that $\begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix} = 0,$ where $x \neq y \neq z$ and p is any real number. (SP 15)

OR

If $\cos 2\theta = 0$, then find the value of $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$. (Exemplar) $\left(\frac{1}{2}\right)$

OR

If $\begin{vmatrix} a & b-x & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, then using properties of determinants, find the value of $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}$ where $x, y, z \neq 0$ (DelhiC 17) (2)

OR

If x ,y and z are unequal and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$ = 0, prove that 1+xyz = 0. (B 17)

OR

Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then find $\lim_{t\to 0} \frac{f(t)}{t^2}$. (Exemplar) (0)

OR

If θ is real number find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$.

(Exemplar) $\left(\frac{1}{2}\right)$

OR

If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix} = -4$ then find the value of $\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}$. (SP 18) (16)
(Hint use $|A| = |adjA|^2$)

OR

If $a+b+c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then using properties of determinants, prove that $a=b=c$. (Bh 15) (MT 15) (AIC 17)

OR

Using properties of determinants, prove that $\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(c+a)^2}{b} \end{vmatrix} = 2(a+b+c)^3$ (SP1 17)

OR

If p≠ 0, q ≠ 0 and $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$ then, using properties of determinants, prove that at least one of the following statements is true: (a) p, q, r are in G. P., (b) α is a root of the equation $px^2+2qx+r=0$. (SP1 17)

3. f(x)= $\begin{cases} \frac{k\cos x}{\pi-2x}, & if\ x \neq \frac{\pi}{2} \\ 3, & if\ x = \frac{\pi}{2} \end{cases}$, find k for which f(x) is continuous.

OR

Discuss the differentiability of the function $f(x) = \begin{cases} 2x-1, & x < \frac{1}{2} \\ 3-6x, & x \geq \frac{1}{2} \end{cases}$ at $x = \frac{1}{2}$

OR

Find 'a' and 'b', if the function given by $f(x) = \begin{cases} ax^2+b, & x<1 \\ 2x+1, & x \geq 1 \end{cases}$ is differentiable at $x=1$.
(SP 18) $(a=1, b=2\quad)$

OR

Show that the function $f(x) = 2x - |x|$, x ϵ R , is continuous but not differentiable at $x=0$.

OR

Determine the value of 'a' and 'b' such that the following function is continuous at $= 0$:

$$f(x) = \begin{cases} \frac{x+\sin x}{\sin(a+1)x}, \text{if} -\pi < x < 0 \\ 2, \quad x = 0 \\ 2\frac{e^{\sin bx}-1}{bx}, \text{if } x > 0 \end{cases}$$

$((a = 0, b$ may be any real number other than zero $))$

OR

For what value of k is the following function continuous at $= -\frac{\pi}{6}$?

$$f(x) = \begin{cases} \frac{\sqrt{3}\sin x+\cos x}{x+\frac{\pi}{6}}, x \neq -\frac{\pi}{6} \\ k, \quad x = -\frac{\pi}{6} \end{cases}$$

OR

If f(x)= $\begin{cases} \frac{1-\cos 4x}{x^2}, \text{when } x < 0 \\ a, \text{when } x = 0 \\ \frac{\sqrt{x}}{\sqrt{(16+\sqrt{x})}-4}, \text{when } x > 0 \end{cases}$ *and f is continuous at* $x = 0$, *find the value of a.*

(AIC 2012), (DelhiC 2013) (8)

OR

If f(x)= $\begin{cases} \frac{\sin(a+1)x+\sin x}{x}, \text{when } x < 0 \\ c, \text{when } x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}}, \text{when } x > 0 \end{cases}$ *and f is continuous at* $x = 0$, *find a, b and c.*

$\left(a = -\frac{3}{2}, b \in R - \{0\}, c = \frac{1}{2}\right)$

4. If $x = a\sin pt, y = b\cos pt$, then show that $(a^2 - x^2)y\frac{d^2y}{dx^2} + b^2 = 0$. (SP 17)

OR

Differentiate $\log\left(\frac{a+b\sin x}{a-b\sin x}\right)$ with respect to x.

OR

Differentiate $\log(x^{\sin x} + \cot^2 x)$ with respect to x. (SP14) $\left(x^{\sin x}\left(\cos x\log x + \frac{\sin x}{x}\right) - 2\cot x\,\mathrm{cosec}^2 x\right)$

OR

If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2y_2 + (x+1)^2y_1 = 2$. (SP 18)

OR

If $y = \log\left[x + \sqrt{x^2+1}\right]$, *prove that* $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

OR

Find $\frac{dy}{dx}$ if $y = x^{x\log x} + (\log x)^x$ $\left(x^{x\log x}.\{(\log x)^2 + 2\log x\} + (\log x)^x\{\log(\log x) + \frac{1}{\log x}\}\right)$

OR

If $x = a\cos\theta + b\sin\theta$ & $y = a\sin\theta - b\cos\theta$, *then prove that* $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.
(AIC 2013), (Delhi 15)

OR

If $x = a(\cos t + \log\tan\frac{t}{2}), y = a(1+\sin t)$, find $\frac{d^2y}{dx^2}$. (Foreign10) $\left(\tan t; \frac{\sec^3 t\tan t}{a}\right)$

OR

Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, *when* $x \neq 0$. (Delhi 2014) $\left(\frac{1}{4}\right)$

OR

If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right), x^2 \le 1$, then find $\frac{dy}{dx}$. (Delhi 15) $\left(\frac{-x}{\sqrt{1-x^4}}\right)$

OR

$If\ \ y = \cos^{-1}[\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}],\ find\ \frac{dy}{dx}$(AIC10) $\left(\frac{-1}{\sqrt{1-x^2}}\right)$

OR

Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right] = \sqrt{a^2-x^2}$

OR

If it is given that for the function $f(x) = x^3 + bx^2 + ax + 5$ on $[1, 3]$, Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$ find the value of a and b. $(a = 11, b = -6)$

5. Find the interval in which the function f given by $f(x) = \sin x - \cos x, 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing. $\left(\left(\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)\right)\right)$

OR

Find the intervals in which the function $f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$ is strictly increasing or strictly decreasing. $(\text{strictly dec in } (-1, 0), \text{strictly inc in } [0, \infty)\ \ )$

OR

Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

OR

Find the equations of the tangent and normal to the curve x =a $\sin^3\theta$, $y = a\cos^3\theta$ at the point where $\theta = \frac{\pi}{4}$. (Delhi 2014) $(T\ \sqrt{2}\,x + \sqrt{2}\,y = a, N\ y = x\ )$

OR

Find the equation(s) of the tangent(s) to the curve $y = (x^3 - 1)(x - 2)$ at the points, where the curve intersects the x-axis. $(3x + y = 3, y = 7x - 14)$

OR

A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the following formula:
$C(x) = x^3 - 45x^2 + 600x$, where x is the number of trees and C(x) is the cost of planting x trees in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one Community Park for a fair distribution. For how many trees should the person place the order so that he has to spend the least amount? How much is the least amount? Use calculus to answer these questions. Which value is being exhibited by the person? (For 20 trees ₹ 2000)

6. Find $\int \frac{(\cos 5x + \cos 4x)}{1 - 2\cos 3x}dx$ $\left(-(\frac{1}{2}\sin 2x + \sin x) + c\right)$

OR

$\int \frac{sec\ x}{1+cosec\ x}dx$ $\left(\frac{1}{4}\log|1+\sin x| + \frac{1}{2}\frac{1}{1+\sin x} - \frac{1}{4}\log|1-\sin x| + c\right)$

OR

Prove that $\int_0^{\pi} x\sin^3 x\ dx = \frac{2\pi}{3}$

OR

Evaluate $\int_0^{\pi/2} \sin 2x\ \tan^{-1}(\sin x)\,dx$ $\left(\frac{\pi}{2} - 1\right)$

OR

Evaluate : $\int e^x(\frac{sin4x-4}{1-cos4x})\,dx$(Delhi 2010) $(e^x.\cot 2x + c)$

OR

Evaluate the following indefinite integral: $\int \frac{\sin\emptyset}{\sqrt{\sin^2\emptyset + 2\cos\emptyset + 3}}d\emptyset$

(SP 16) $\left(-\sin^{-1}\frac{\cos\emptyset-1}{\sqrt{5}}+c\right)$

OR

Evaluate: $\int(x-3)\sqrt{x^2+3x-18}\,dx$. (Delhi 2014)

$$\left(\frac{1}{3}(x^2+3x-18)^{\frac{3}{2}}-\frac{9}{2}\left[\frac{(2x+3)}{4}\sqrt{x^2+3x-18}+\frac{81}{8}\log\left|x+\frac{3}{2}+\sqrt{x^2+3x-18}\right|\right]+c\right)$$

Evaluate $\int\frac{x^2}{x^4+x^2-2}dx$ (C 15) $\left(\frac{2}{3\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}+\frac{1}{3}\,log\,\left(\left|\frac{x-1}{x+1}\right|\right)+c\right)$

OR

Find : $\int\frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}dx$ (Foreign 16) $\left(x+\frac{1}{4\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+\frac{27}{8\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|+c\right)$

OR

Find : $\int\frac{x\,dx}{(2+x^2)(4+x^4)}$ (Foreign 17) $\left(\frac{1}{16}\log|2+x^2|-\frac{1}{32}\log|4+x^4|+\frac{1}{16}\tan^{-1}\left(\frac{x^2}{2}\right)+c\right)$

OR

Find : $\int\frac{e^x\,dx}{(2+e^x)(4+e^{2x})}$ (Foreign 17) $\left(\frac{1}{8}\log|2+e^x|-\frac{1}{16}\log|4+e^{2x}|+\frac{1}{8}\tan^{-1}\left(\frac{e^x}{2}\right)+c\right)$

OR

Find: $\int\frac{1-\sin x}{\sin x(1+\sin x)}dx$ (SR 16) $(\log|cose\ x-\cot x\,|-2\tan x+2\sec x+c)$

OR

Evaluate $\int_0^{\pi/2}\frac{dx}{1+\sqrt{tan\,x}}$ (*DelhiC* 17) $\left(\frac{\pi}{4}\right)$

OR

Evaluate $\int_{-2}^{2}\frac{x^2}{1+5^x}dx$ (NR 16) $\left(\frac{8}{3}\right)$

OR

Evaluate $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$ (Delhi 17) $\left(\frac{\pi^2}{4}\right)$

OR

Evaluate the following definite integral: $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$ (SP 16) (π^2)

OR

Evaluate: $\int_0^1\cot^{-1}(1-x+x^2)\,dx$ (SR 16) $\left(\frac{\pi}{2}-\log 2\right)$

7. Solve the differential equation $x\frac{dy}{dx}=y(log\,y-log\,x+1)$ $((y=xe^{-cx}))$

OR

Show that the differential equation $xdy-ydx=\sqrt{x^2+y^2}$ dx is homogeneous, and solve it. (SP 14) $(y+$ $x2+y2=cx2$

OR

Solve the following differential equation:

$\frac{dy}{dx}+y\cot x=4x\,cosec\,x, given\ that\ y=0\ when\ x=\frac{\pi}{2}$.

OR

Find the particular solution of the differential equation :

$ye^ydx=(y^3+2xe^y)dy, y(0)=1$ $\left(\text{IF}=\frac{1}{y^2}, x=-y^2e^{-y}+\frac{y^2}{e}\right)$

OR

Find the general solution of the differential equation $xlogx\frac{dy}{dx}+y=\frac{2}{x}logx$ (Delhi 2010)
$(I.F=\log x, xylog\,x=-2\log x-2+cx\,)$
OR
Solve the differential equation: $(\sin^{-1}y-x)dy=(\sqrt{1-y^2})dx$, find the solution curve passing through the point (1,0). $\left(x=(\sin^{-1}y-1)+2e^{-\sin^{-1}y}\right)$
OR
Find the differential equation of the family of all circles touching the y-axis at the origin. (DelhiC 2010)
$\left(2xy\frac{dy}{dx}+x^2-y^2=0\right)$

8. If $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$, $\vec{b}=\hat{\jmath}-\hat{k}$ then find a vector $\vec{c}$ such that $\vec{a}X\,\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$. $\left(\frac{5}{3}\hat{\imath}+\frac{2}{3}\hat{\jmath}+\frac{2}{3}\hat{k}\right)$
OR

If $\vec{a}=\hat{\imath}+4\hat{\jmath}+2\hat{k}$, $\vec{b}=3\hat{\imath}-2\hat{\jmath}+7\hat{k}$ and $\vec{c}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}=18$.(AI 2010) $(\lambda=2{,}64\,\hat{\imath}-2\hat{\jmath}-28\hat{k}\,)$

OR

If the sum of 2 unit vectors is a unit vector then find the magnitude of their difference and angle between them. (DelhiC 2012) $\left(\sqrt{3};\ \frac{2\pi}{3}\right)$

OR

Given that vectors $\vec{a},\vec{b},\vec{c}$ form a triangle such that $\vec{a}=\vec{b}+\vec{c}$. Find p, q, r, s such that area of triangle is $5\sqrt{6}$ where $\vec{a}=p\hat{\imath}+q\hat{\jmath}+r\hat{k}$, $\vec{b}=s\hat{\imath}+3\hat{\jmath}+4\hat{k}$ and $\vec{c}=3\hat{\imath}+\hat{\jmath}-2\hat{k}$. $\left((p=-8{,}8, q=4, r=2 \text{ and } s=-11{,}5)\right)$
OR
The magnitude of the vector product of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to $\sqrt{2}$. Find the value of $\boldsymbol{\lambda}$. (1)
OR
If $\vec{a}=2\hat{\imath}-\hat{\jmath}-2\hat{k}$ and $\vec{b}=7\hat{\imath}+2\hat{\jmath}-3\hat{k}$ then express $\vec{b}$ in the form of $\vec{b}=\vec{b}_1+\vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$. (AI 17) $\left(\lambda=2,(4\hat{\imath}-2\hat{\jmath}-4\hat{k})+(3\hat{\imath}+4\hat{\jmath}+\hat{k})\right)$
OR
If $\vec{a},\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3, |\vec{b}|=5$ and $|\vec{c}|=7$. Find the angle between $\vec{a}$ and $\vec{b}$. (Delhi 2014) $\left(\frac{\pi}{3}\right)$
OR
If $\vec{a},\vec{b},\vec{c}$ are 3 mutually perpendicular vectors of the same magnitude, prove

that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with the vectors $\vec{a},\vec{b}$ and $\vec{c}$ also find the angle. (NCERT) (DelhiC 2011, 13)

OR

If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then prove that (i)
$\vec{a}=\pm2(\vec{b}\,\mathrm{X}\,\vec{c})$ (ii) $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=\pm1$
OR
Prove that $\vec{a}.\{(\vec{b}+\vec{c})X(\vec{a}+2\vec{b}+3\vec{c})\}=[\vec{a}\ \ \vec{b}\ \ \vec{c}]$.
OR
If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{o}$, then prove that $\vec{a}X\,\vec{b}=\vec{b}X\vec{c}=\vec{c}X\vec{a}$, and hence show that $[\vec{a}\ \vec{b}\ \vec{c}]=0$
OR

If the vectors $\vec{p} = a\hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{q} = \hat{\imath} + b\hat{\jmath} + \hat{k}$ and $\vec{r} = \hat{\imath} + \hat{\jmath} + c\hat{k}$ are coplanar, then for $a, b, c \neq 1$, show that $\frac{1}{1-a} = \frac{1}{1-b} = \frac{1}{1-c} = 1.$

9. Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{1}$ and $\frac{7-7x}{3p} = \frac{5-y}{1} = \frac{11-z}{7}$ are at right angles.

OR

Find the equation of the plane passing through the points (1, 2, 3), (0, -1, 0) and parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$ $(6x - 3y + z = 3)$

OR

Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$ are coplanar. Also, find the plane containing these lines. $(x - 2y + z = 0)$

OR

Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$.

Also, find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not.

(SR 16) $(8x + y - 5z = 7,\ \ contains)$

OR

A plane meets the coordinate axes in A, B and C such that the centroid of ΔABC is the point (α, β, γ) . Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

OR

A plane meet the x,y and z axes at A,B and C respectively, such that the centroid of the triangle ABC is (1,-2,3). Find the vector and Cartesian equation of the plane. (SP14)

$\left(\vec{r}.\left(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}\right) = 18; 6x - 3y + 2z - 18 = 0\ \right)$

OR

Find the equation of the line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

And passes through the point (1, 1, 1). $\left(\frac{x-1}{3} = \frac{y-1}{10} = \frac{z-1}{17}\right)$

OR

Find the equation of plane passing through the points A(3, 2, 1), B(4, 2, -2) and C(6, 5 -1) and hence find the value of λ for which A(3, 2, 1), B(4, 2, -2), C(6, 5 -1), D(λ, 5, 5) are coplanar. (SR 16) $(9x - 7y + 3z - 16 = 0; \lambda = 4$

OR

Find the equation of the plane passing through the point (3, 2, 0) and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ (Foreign 15) $(x - y + z = 1)$

OR

Find the values of 'a' so that the following lines are skew: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}$; $\frac{x-4}{5} = \frac{y-1}{2} = z$

OR

Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:

$\vec{r} = (8 + 3\lambda)\hat{\imath} - (9 + 16\lambda)\hat{\jmath} + (10 + 7\lambda)\hat{k}$

$\vec{r} = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k} + \mu\,(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$ (SP2 17) $(14\ units)$

OR

Find the shortest distance between the following pair of lines and hence write whether the lines are intersecting or not :

$\frac{x-1}{2}=\frac{y+1}{3}=z;\ \frac{x+1}{5}=\frac{y-2}{1};z=2\ .$ (Foreign10)

$\left(\left|\frac{-9}{\sqrt{(-1)^2+(5)^2+(-13)^2}}\right|=\frac{9}{\sqrt{195}}\text{ units ; do not intersect}\right)$

OR

Find the image of the point (2,-1,5) in the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$. Also write the equation of the line joining the given point and its image and find the length of the segment joining the given point and its image. (AIC 2013) $(\lambda=-1, \text{FP}(1,2,3), \text{Image }(0,5,1), \text{length}=2\sqrt{14}\text{ units })$

OR

Find the position vector of the foot of the perpendicular and the perpendicular distance from the point P with position vector $2\hat{\imath}+3\hat{\jmath}+4\hat{k}$ to the plane $\vec{r}.(2\hat{\imath}+\hat{\jmath}+3\hat{k})-26=0$. Also, find image of P in the plane. (CR 16)

$\left(\lambda=\frac{1}{2}, FP=3\hat{\imath}+\frac{7}{2}\hat{\jmath}+\frac{11}{2}\hat{k};\ dist=\frac{\sqrt{14}}{2}\ ;\ Image=(4,4,7)\right)$

10. Find the mean number of heads in three tosses of a fair coin.

OR

Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of diamond cards drawn. Also, find the mean and the variance of the distribution. $\left(Mean=\frac{1}{2}, \text{Var}=\frac{3}{8}\right)$

OR

Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution. $\left(\text{M}=\frac{2}{5}, \text{V}=\frac{144}{475}\right)$

OR

A bag contains (2n+1) coins. It is known that n of these coins have a head on both sides where as the rest of the coin are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, find the value of n.

OR

For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9\,\text{P}(\text{X}=4)=\text{P}(\text{X}=2)$. Find the probability of success. $\left(8p^2+2p-1=0,\ p=\frac{1}{4}\right)$ (RU 15)

OR

A person wants to construct a hospital in a village for welfare. The probabilities are 0.4 that some bad element oppose this work, 0.8 that the hospital will be completed, if there is no any oppose of any bad element and 0.3 that the hospital will be completed, if bad element oppose. Determine the probability that the construction of hospital will be completed. $\left(\frac{3}{5}\right)$

11. A shopkeeper sells 3 types of flower seeds A_1,A_2 and A_3. They are sold as a mixture where the proportion are 4:4:2 respectively. The germination rates of 3 types of seeds are 45%, 60% and 35%. Calculate the probability.
    (a) Of a randomly chosen seed to geminate.
    (b) That it is of the type A_2, given that a randomly chosen seed does not germinate. (SP14)

    $\left(0.49, \frac{16}{51}\right)$ (NCERT Exemplar)

A factory has two machines A and B. Past record shows that Machine A produced 60% of the items of output and machine B produced 40% of the items. Further 2% of the items produced by machine A and only 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

OR

Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they come from Bag III. $\left(\frac{64}{199}\right)$

OR

Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred from bag I to Bag II and then 2 balls are drawn at random (without replacement) from bag II. The balls so drawn are found to be both red in colour. Find the probability that the transferred ball is red. (SP 14, G 15) $\left(\frac{5}{9}\right)$

OR

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be hearts. Find the probability of the lost card being of hearts. (DelhiC 2012) $\left(\frac{11}{50}\right)$

OR

If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup. (0.95)

OR

There are 3 coins. One is 2 headed coin another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tail 40% of the times. One of the 3 coins is chosen at random and tossed, and it shows heads, what is the probability that it was the 2 headed coin? $\left(\frac{20}{47}\right)$ (AI 2014)

OR

A letter is known to have come from either TATANAGAR or CALCUTTA . On the envelop just 2 consecutive letters TA are visible. What is the probability that letter has come from TATANAGAR ? $\left(\frac{7}{11}\right)$ (NCERT Exemplar)

OR

An urn contains 3 red and 5 black balls. A ball is drawn at random, its colour is noted and returned to the urn. Moreover, 2 additional balls of the colour noted down, are put in the urn and then two balls are drawn at random (without replacement) from the urn. Find the probability that both the balls drawn are of red colour. (B 15) $\left(\frac{1}{8}\right)$

For Section D(6marks)

12. Ten students were selected from a school on the basis of values for giving awards and were divided into three groups. First group comprises hard workers, second group comprises honest and law abiding students and third group contains vigilant and obedient students. Double the number of students of the first group added to the number in second group gives 13, while the combined strength of the first and second group is 4 times that of the third group using matrix method, find the number of students in each group.

$$\frac{1}{10}\begin{pmatrix}-8 & 10 & -2\\ 16 & -10 & 4\\ 2 & 0 & -2\end{pmatrix}, x=5, y=3, z=2$$

OR

Using elementary transformations, find the inverse of the matrix A= $\begin{pmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{pmatrix}$ and use it to solve the following system of linear equations:

$8x+4y+3z=19$

$2x+y+z=5$
$x+2y+2z=7$

(Delhi 16) $\left(\begin{pmatrix} 0 & \frac{2}{3} & -\frac{1}{3} \\ 1 & -\frac{13}{3} & \frac{2}{3} \\ -1 & 4 & 0 \end{pmatrix}; x=1, y=2, z=1\right)$

OR

If A = $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ $and\ B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB use this to solve the following system of equations:

$x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$
(AIC10), (AIC 2012) (F 17) $(6I,\ x = 2, y = -1, z = 4)$

13. Show that the height of the right circular cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm. Also find its volume. $\left(\frac{2048\pi}{3}\ cm^3\right)$

OR

Prove that the volume of the largest cone that can be in inscribed in a sphere of radius R, is $\frac{8}{27}$ of the volume of the sphere. (DelhiC 2010), (Foreign 2013),(AIC 2013)

OR

Show that the isosceles triangle of maximum area that can be inscribed in a given circle is an equilateral triangle. Also find its area. $\left(side = \sqrt{3}r,\ A = \frac{3\sqrt{3}}{4}r^2\right)$

OR

Prove that the height of right circular cylinder of maximum volume that can be inscribed in a given cone of height h is h/3.

OR

Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius 'a' is a square of side $\sqrt{2}a$.

OR

A wire of length 36 m is to be cut into 2 pieces. One of the 2 pieces is to be made into a square and the other into a circle. What should be the lengths of 2 pieces so that the combined area of circle and square is minimum ?(AIC10) $\left(\frac{144}{\pi+4}\ m, \frac{36\pi}{\pi+4}\ m\right)$

OR

If the lengths of 3 sides of a trapezium other than base are each equal to 10 cm, then find the area of the trapezium when it is maximum. (AI 2010),(DelhiC 2013) $\left(\frac{75\sqrt{3}}{2}\ cm^2\right)$

14. Evaluate $\int_0^{\pi/2} \frac{xsinxcosx}{\sin^4 x+\cos^4 x} dx$. (AIC10), (Delhi 2014), (SP 17) $\left(\frac{\pi^2}{16}\right)$

OR

Evaluate $\int \frac{sin\ x}{\sin^3 x+\cos^3 x}\ dx$ (SP 18) $\left(-\frac{1}{3}\log|\tan x + 1| + \frac{1}{6}\log|\tan^2 x - \tan x + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + c\right)$

OR

Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-cos\ 2x} dx$ (SP 18) $\left(\frac{\sqrt{3}\pi^2}{18}\right)$

OR

Evaluate: $\int_0^{\pi} \frac{xtanx}{secx+tanx} dx$

OR

Prove that $\int_0^{\pi/2} log(tan\ x + cot\ x)dx = \pi log\ 2$

OR

Evaluate: $\int_{-2}^{2}|x\cos\frac{\pi x}{2}|\,dx.$

OR

Evaluate $\int_1^3(3x^2+2x)dx$ as limit of sums. (Delhi 2010) (34)

15. Using integration, find the area of the region bounded by the lines
$4x-y+5=0;\ x+y-5=0\ and\ x-4y+5=0.$ (Foreign10) $\left(\frac{15}{2}\ Sq.units\right)$

OR

Using integration find the area bounded by the tangent to the curve $4y=x^2$ at the point $(2,1)$ and the lines whose equations are $x=2y$ and $x=3y-3$. $(1\ squ)$

OR

Using integration, find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$.
(AI 2010) $\left(\frac{\sqrt{2}}{6}+\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}\ Sq.units\right)$

OR

Using integration, find the area in the first quadrant bounded by the curve $y=x|x|$, the circle $x^2+y^2=2$ and the y-axis. (SP 18) $\left(\left(\frac{1}{6}+\frac{\pi}{4}\right)\ squ\right)$

OR

Using integration, find the area of the region bounded by the curve $x^2+y^2=4$ and $(x-2)^2+y^2=4$. (AIC10,12,13) $\left(\frac{8\pi}{3}-2\sqrt{3}\ Sq.units\right)$

16. Find the coordinate of the point where the line through (3,-4,-5) and (2,-3,1) crosses the plane determined by points A(1,2,3), B(2,2,1) and C(-1,3,6). (Foreign10)

$$\left(\text{Equation of plane}\begin{vmatrix}x-1 & y-2 & z-3\\ 1 & 0 & -2\\ -2 & 1 & 3\end{vmatrix}=0,\lambda=2,\text{Point of int. is }(1,-2,7)\right)$$

OR

Find the equation of the plane through the point (4, -3, 2) and perpendicular to the line of intersection of the planes $x-y+2z-3=0$ and $2x-y-3z=0$. Find the point of intersection of the line $\vec{r}=\hat{\imath}+2\hat{\jmath}-\hat{k}+\lambda(\hat{\imath}+3\hat{\jmath}-9\hat{k})$ and the plane obtained above. (SP 17) $(5x+7y+z=1,\lambda=-1;-\hat{\jmath}+8\hat{k})$

OR

If lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines. (Delhi 15) $\left(k=\frac{9}{2};\ -5x+2y+z+6=0\right)$

OR

Find the distance of the point (3,4,5) from the plane $x+y+z=2$ measured parallel to the line $2x=y=z$. (DelhiC 2012) $(\lambda=-4,\text{Point of intersection is }(1,0,1),\text{distance }=6\text{ unit})$

OR

Find the distance of the point $-2\hat{\imath}+3\hat{\jmath}-4\hat{k}$ from the line $\vec{r}=\hat{\imath}+2\hat{\jmath}-\hat{k}+\lambda(\hat{\imath}+3\hat{\jmath}-9\hat{k})$ measured parallel to the plane: $x-y+2z-3=0$. $\left(\frac{\sqrt{59}}{2}\right)$

OR

Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ and twice of its y-intercept is equal to three times its z-intercept. (DelhiC 17) $(\lambda=-1,x+2y+3z=4$

17. A firm has to transport atleast 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is ₹ 400 and each small van is ₹ 200. Not more than ₹ 3,000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP and solve it graphically given that the objective is to minimize cost.

OR

A company produces two different products. One of them needs ¼ of an hour of assembly work per unit, 1/8 of an hour in quality control work and ₹ 1.2 in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and ₹ 0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of 90 hours available for assembly and 80 hours for quality control. The first product described has a market value (sale price) of ₹ 9 per unit and the second product described has a market value (sale price) of ₹ 8 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit.

OR

If a young man rides his motor-cycle at a speed of 25 km/hr, he has to spend ₹ 2 per km on petrol with very little pollution in the air. If he drives his motor-cycle r at a speed of 40 km/hr, the petrol cost increases to ₹ 5 per km and rate of pollution also increases. He has a maximum of ₹ 100 to spend on petrol and travel a maximum distance in one hour time. Express this problem as an LPP and solve it graphically.

What value is indicated in this question? (DelhiC 2014)

$\left(\text{Max Distance } = 30 \text{ km at } \left(\frac{50}{3}, \frac{40}{3}\right)\right)$;Indication of Value: Vehicle should be driven at a moderate speed to decrease the pollution.

OR

An aeroplane can carry a maximum of 200 passengers. A profit of ₹500 is made on each executive class ticket out of which 20% will go to the welfare fund of the employees. Similarly a profit of ₹ 400 is made on each economy class ticket out of which 25% will go for the improvement of facilities provided to economy class passengers. In both cases, the remaining profit goes to the airline's fund. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. Make the above as an LPP and solve graphically. Do you think, more passengers would prefer to travel by such an airline than by others? (Max profit = ₹64,000 at x=40,y=160.) (Foreign 2013)