CBSE Sample Paper

CLASS – XII

Write Your Roll No.

ANNUAL EXAMINATION 2020
MATHEMATICS - 041

Max Time 3 Hrs | Max Marks 80

General instructions:

1. All the questions are compulsory.
2. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
3. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
4. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

Section A

Question numbers 1 to 10(carry 1 mark each) are multiple choice type questions. Select the correct option:

1. $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x is equal to:

   (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{\sqrt{3}}{2}$

2. Projection vector of $\vec{a}$ on $\vec{b}$ is

   (A) $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ (B) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ (C) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (D) $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

3. If A is a square matrix, then $A^TA + AA^T$ is:

   (A) Unit matrix (B) null matrix (C) symmetric matrix (D) skew-symmetric matrix

4. If A and B are invertible matrices of order 3, $|A| = 2$ and $\left|(AB)^{-1}\right| = -\frac{1}{6}$. Find $|B|$.

(A) -3/2 (B) 3 (C) -3 (D) None of these

5. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is
(A) $\frac{2}{5}$ (B) $\frac{1}{15}$ (C) $\frac{8}{15}$ (D) $\frac{4}{15}$

6. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{7}$

7. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

(A) p = q (B) p = 2q (C) q = 2p (D) q = 3p

8. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

(A) $\tan x + \cot x + c$ (B) $(\tan x + \cot x)^2 + c$

(C) $\tan x - \cot x + c$ (D) $(\tan x + \cot x)^2 + c$

9. The perpendicular distance of the point P (1, 2, 3) from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$

(A) 7 (B) 5 (C) 0 (D) None of these

10. The equation of the line in vector form passing through the point(−1,3,5) and parallel to line $\frac{x-3}{2} = \frac{y-4}{3}; z = 2$ is

(A) $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$

(B) $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath})$

(C) $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) + \lambda(-\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$

(D) $\vec{r} = (2\hat{\imath} + 3\hat{\jmath}) + \lambda(-\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$

Question numbers 11 to 15(carry 1 mark each) are to be filled in the blanks:

11. The reason for the relation R in the set {1,2,3} given by R={(1,2),(2,1)} not to be transitive is

____

12. The value of 'k' for which the following function

$F(x) = \begin{cases} \frac{(x+3)^2-36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ **is continuous at $x = 3$ is _________**

13. The value of x for which, matrix $\begin{bmatrix} 2(x+1) & 2x \\ x & x-2 \end{bmatrix}$ is singular _________

14. If tangent to the curve $y^2 + 3x - 7 = 0$ at the point (h, k) is parallel to line x–y = 4, then value of k is _________?

OR

For the curve = $5x - 2x^3$, if increases at the rate of 2 units/sec, then at $x = 3$ the slope of the curve is changing at _________

15. What is the cosine of the angle which the vector $\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k}$ makes with y axis?

OR

Write the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\,\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$.

(Question numbers 16 to 20 carry 1 mark each) Answer the following questions:

16. **If A = $\begin{pmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{pmatrix}$, then for what value of α is A is an identity matrix?**

17. **Evaluate : $\int \cos^{-1}(\sin x)dx$**

18. **Evaluate $\int \frac{3x}{3x-1}dx$**

19. **Evaluate $\int_0^{\pi/2} e^x(\sin x - \cos x)\, dx$**

OR

Evaluate: $\int_0^1 \tan^{-1} \frac{2x-1}{1+x-x^2} dx$

20. **Write the general solution of differential equation $\frac{dy}{dx} = e^{x+y}$**

Section B

Question numbers 21 to 26 carry 2 marks each.

21. Find the value of $\sin^{-1}\frac{2}{3} + \sec^{-1}\frac{3}{2}$.

OR

If f : R→ R defined as f(x) = $\frac{2x-7}{4}$ is an invertible function, write $f^{-1}(x)$.

22. Differentiate $\tan^{-1}\left(\frac{\sin x + \cos x}{\sin x - \cos x}\right)$ with respect to x.

23. Show that the function f(x) = $x^3 - 3x^2 + 9x + 5$ is strictly increasing on R.

24. Using vectors, prove that the points (2, – 1, 3), (3, – 5, 1) and (– 1, 11, 9) are collinear.

OR

Find the area of the parallelogram whose diagonals are represented by the vectors

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

25. Find the Cartesian and Vector equations of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$.

26. If P(not A) = 0·7, P(B) = 0·7 and P(B/A) = 0·5, then find P(A/B).

Section C

Question numbers 27 to 32 carry 4 marks each.

27. Let Z be the set of all integers and R be the relation on Z defined as R={(a,b): a,b ∈ Z, and (a-b) is divisible by 5}. Prove that R is an equivalence relation.

28. If $y = \log\left[x + \sqrt{x^2 + a^2}\right]$, *then prove that* $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

OR

Let $f(x) = x - |x - x^2|, x \in [-1, 1]$. Find the point of discontinuity, (if any), of this function on $[-1, 1]$.

29. Evaluate $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

30. **Find the particular solution of the differential equation:**

$$x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right),\ x \neq 0, when\ x = 1, y = \frac{\pi}{2}.$$

31. **An aeroplane can carry a maximum of 250 passengers. A profit of ₹ 1,500 is made on each executive class ticket and a profit of ₹ 1,000 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 3 times as many passengers prefer to travel by economy class than by executive class. Frame the Linear Programming Problem to determine how many tickets of each type must be sold in order to maximize the profit for the airline.**

32. **Bag A contains 3 red and 2 black balls, while bag B contains 2 red and 3 black balls. A ball is drawn at random and transferred to bag B and then a ball is drawn from bag B at random. If this ball was found to be red ball, find the probability that the ball drawn from bag A was red.**

OR

The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and

$P(X = 2) = P(X = 3)$ such that $\sum p_i x_i^2 = 2\sum p_i x_i$, find the value of p.

Section D

Question numbers 33 to 36 carry 6 marks each.

33. **If A= $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations:**

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2; \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5; \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$$

OR

Using properties of determinants. Show the following:

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

34. **Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.**

OR

Find the equation of tangent to the curve x = a cos θ + a θ sin θ, y = a sin θ – a θ cos θ at any point θ of the curve. Also show that at any point θ of the curve the normal is at a constant distance from origin.

35. **Using integration, find the area of the triangle ABC, coordinates of whose vertices are A(4,1), B(6,6) and C(8,4).**

36. **Show that the lines**

$$\frac{x-1}{2}=\frac{y-3}{4}=\frac{z}{-1} \text{ and } \frac{x-4}{3}=\frac{1-y}{2}=z-1$$

are coplanar. Hence find the equation of the plane containing these lines.

Answer Key and hints

Set 1

Section A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	14(or)	15	15(or)
B	A	C	C	C	D	D	C	A	B		12	-2	-3/2	Dec 72 u/s		

Section B

21(or). $\{4,-4\}, \Phi$ 22. $-\frac{1}{2}$ 23. $\left(1,\frac{1}{2}\right)$ **24. 3** **25.** 26. $\left(\frac{3}{14}\right)$

Section C

29. $\left(5\sqrt{x^2+4x+10}-7\log\left|(x+2)+\sqrt{x^2+4x+10}\right|+c\right)$ **30.** **31.** $(Max\ P = 1500x + 1000y;\ x+y\le 250, x\ge 25, y\ge 3x, x\ge 0, y\ge 0)$ **32.** $\left(\frac{9}{13}\right)$ **32(or)** $\left(\frac{5}{24}\right)$

Section D

33. $\left(A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}; \ x = 2, y = -3, z = 5\right)$