TARUN CLASSES OF MATHEMATICS

MATHEMATICS –(041)

Class XII – 2019-2020

MAX. MARKS : 80 DURATION : 3HOURS

SECTION –A

Q1 - Q10 are multiple choice type questions. Select the correct option

1) If A and B are invertible matrices of order 3, |A| = 2 and $|(AB)^{-1}| = -\frac{1}{6}$. Then |B| is :

a) $-\frac{3}{2}$ b) 3 c) - 3 d) None of these

2) If A is matrix of order m × n and B is a matrix such that AB' and B'A are both defined, then order of matrix B is

a) m × m b) n × n c) n × m d) m × n

3) The two *j + k & 3 i -j + 4k* vectors represents the two sides AB and AC, respectively of a ΔABC. The length of the median through A is :

a) $\frac{\sqrt{34}}{2}$ b) $\frac{\sqrt{48}}{2}$ c) $\sqrt{18}$ d) None of these

4) If P(B) = $\frac{3}{5}$, P($\frac{A}{B}$) = $\frac{1}{2}$ & P(AUB) = $\frac{4}{5}$, Then P(AUB)' + P(A'UB) is equal to

a) $\frac{1}{5}$ b) $\frac{4}{5}$ c) 1 d) $\frac{1}{2}$

5) Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let Z = px+qy, where p, q > 0. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is

a) p = 2q b) p = 2 q c) p = 3q d) p = q

6) The greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ are respectively :

a) $\frac{5\pi^2}{4}$ & $\frac{\pi^2}{8}$ b) $\frac{\pi}{2}$ & $-\frac{\pi}{2}$ c) $\frac{\pi^2}{4}$ & $-\frac{\pi^2}{4}$ d) None of these

7) For the following probability distribution:

X	1	2	3	4
P(X)	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ is equal to :

a) 3 b) 5 c) 7 d) 10

8) If $\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx$ = = ax + b log $|4e^x + 5e^{-x}|$ + C, then

a) a = $\frac{-1}{8}$, b = $\frac{7}{8}$ b) a = $\frac{-1}{8}$, b = $\frac{-7}{8}$ c) a = $\frac{1}{8}$, b =- $\frac{7}{8}$ d) a = $\frac{1}{8}$, b = $\frac{7}{8}$

9) The co-ordinates of the foot of the perpendicular drawn from the origin to a plane is (2, 4, –3). The equation of the plane is :

a) 2x- 4y - 3z = 29 b) 2x- 4y +3z = 29 c) 2 x + 4y - 3z = 29 d) None of these

10) The plane 2x – 3y + 6z – 11 = 0 makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is equal to:

a) $\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{2}}{3}$ c) $\frac{2}{7}$ d) $\frac{3}{7}$

11) If 'f' be the greatest integer function & 'g' be an absolute value function , find the value of (fog)($\frac{-3}{2}$) + (gof)($\frac{4}{3}$) .

12) Find the value of k , so that the function f is continuous at x = 2.

f(x) = $\begin{cases} \frac{2^{x+2}-16}{4^x-16} & , if\ x \neq 2 \\ k \quad , & if\ x = 2 \end{cases}$

13) On using elementary column operations $C_2 \rightarrow C_2 - 2C_1$ in the following matrix equation, Write new Matrix equation .

$$\begin{bmatrix} 1 & 3 \\ -4 & 7 \end{bmatrix} = : \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ -3 & -2 \end{bmatrix}$$

14) For the curve $y = 5x-2x^3$, if x increases at the rate of 3 units/sec, then how fast is the slope of the curve changing when x=3 is……..

OR

The curves $y = 4x^2 + 2x - 8$ and $y = x^3 - x + 13$ touch each other at the point_____.

15) Vector of magnitude 5 units and in the direction opposite to $2\hat{i} + 3\hat{j} - 6k$ is___ OR

The vectors a=3i-2j+2k and b = -i -2k are the adjacent sides of a parallelogram, Then angle between its diagonals…….

16) If *a*, *b* and *c* are non zero numbers, then $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$, Write Value of determinant & Justify.

17) $\int_0^{\Pi/2} \sqrt{1 - \sin 2x}\ dx$ is equal to …….

18) $\int_0^a \frac{1}{4+x^2}\ dx = \Pi/8$, find the value of 'a' . OR $\int \frac{x+3}{(x+4)^2} e^x\ dx$ = ……

19) Find $\int x\, e^{(1+x^2)} dx$.

20) The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents a family of……..

What is the order of the differential equation of all circles of given radius a

SECTION –B

21) Let S is the set of all real numbers & R is the relation in S , defined by R = { (a,b): $a \leq b^3$} . OR

Evaluate : $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$.

22) Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

23) Prove that for any three non zero vectors a, b , c , [a-b b-c c-a] = 0. OR
Find a vector of magnitude of 9 which is perpendicular to both the vectors 4i –j +3k & -2i + j – 2k .

24) Find the shortest distance b/w the following lines : (x-3)/1 = (y-5)/-2 = (z-7)/1 & (x+1)/7 = (y+1)/-6 = (z+1)/1 .Also find the equation of the shortest distance.

25) A & B throw a pair of dice turn by turn . A wins the game if he gets a total of 9 & B wins the game if he gets total of 7 . If A starts the game , Find the probability that A wins the game.

26) Determine the intervals in which f(x) = sin x - cos x , $0 \leq x \leq 2$ pi is strictly increasing or strictly decreasing .

SECTION –C

27) Show that f : R →{xϵR : -1 <x<1] defined by $f(x) = \frac{x}{1+|x|}$, is one one & onto function.

28) If x = a(cos 2θ + 2θ sin 2θ) and y = a(sin 2θ − 2θ cos 2θ) , find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$. OR

If $\sqrt{1-x^6}+\sqrt{1-y^6}$ = a (x^3- y^3) , Prove that : $\frac{dy}{dx}=\frac{x^2\sqrt{1-y^6}}{y^2\sqrt{1-x^6}}$.

29) Solve the differential equation x dy − y dx = $\sqrt{x^2+y^2}$ dx .

30) $\int_{-1}^{3/2}|x\ sin\Pi x|\ dx$

31) Two numbers are selected at random (Without replacement) from the positive integers 2,3,4,5,6,& 7. Let X denote the larger of the two numbers obtained . Find the mean & Variance of the probability distribution of X .

OR

Bag I contains 3 red & 4 black balls & Bag II contains 4 red & 5 black balls . Two balls are transferred from first bag to the second bag ,& then a ball is drawn from bagII & Red in colour. Find the probability that the transferred balls are both Red .

32) An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

SECTION – D

33) If A+B+C = Π, Show that:

$$\begin{vmatrix} sin^2A & \sin A\cos A & cos^2A \\ sin^2B & \sin BcosB & cos^2B \\ sin^2C & \sin C\ cosC & cos^2C \end{vmatrix} = \text{-sin (A-B) sin (B-C) sin (C-A)}$$

OR

Compute A^{-1} for the matrix A = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, Using A^{-1} , solve system of equations :

x - 2 y = 10 , 2x – y - z = 8 & -2y + z = 7

34) Using integration , find the area of the following region , $\{(x,y): |x-1|\leq y\leq\sqrt{5-x^2}\}$.

35) Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle. **OR**

Find the maximum area of the isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 with its vertex at one end of major axis .

36) i) Find the equation of a plane passing through the points A(2,1,2) and B(4,-2,1) and perpendicular to the plane r(i-2k)=5. Also, find the coordinates of the point where the line passing through the points (3,4,1) and (5,1,6) crosses the plane thus obtained.

ii) The plane 4x + 7y + 4z + 81 =0 is rotated through a right angle about its line of intersection with the plane 5x + 3y + 10z = 25. Write equation of the plane in its new position.

''WELL DONE IS BETTER THAN WELL SAID , MAN IS GREAT BY DEEDS NOT BY BIRTH''

TARUN SHARMA