

TARUN CLASSES OF MATHEMATICS

MATHEMATICS –STANDARD (041)

Class X – 2019-2020

MAX. MARKS : 80

DURATION : 3HOURS

SECTION –A

Q 1- Q 10 are multiple choice questions. Select the most appropriate answer from the given options.

- 1) Decimal representation of $\frac{246}{2^8 \times 5^{-3}}$ will be :
 a) Terminate after 6 decimal place b) Terminate after 7 decimal place c) not terminate d) can't determined
- 2) Consider the data:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	3	12	27	57	75	80

The modal class is:

- a) 10-20 b) 20-30 c) 30-40 d) 50-60
- 3) The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is
 a) 13 b) 65 c) 875 d) 1750
- 4) The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is
 a) 3 b) -3 c) -12 d) no value
- 5) If triangle ABC is right angled at C, then the value of $\sec(A+B)$ is :
 a) 0 b) 1 c) Not Defined d) $\frac{2}{\sqrt{3}}$
- 6) If $\cot \theta = \frac{15}{8}$, Then $\frac{(2+2 \sin \theta)(1-\sin \theta)}{(1+\cos \theta)(2-2 \cos \theta)}$ equals to :
 a) $\frac{225}{64}$ b) $\frac{64}{225}$ c) $\frac{289}{64}$ d) $\frac{64}{289}$
- 7) Evaluate : $\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ$:
 a) 3 b) 2 c) 1 d) 0
- 8) If A(4, -3), B(3, -2) and C(2, 8) are vertices of a triangle, then the distance of it's centroid from the y-axis is :
 a) $\sqrt{10}$ b) 1 c) 3 d) $\frac{9}{2}$
- 9) Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment the joining A(2, -2) and B(3, 7) :
 a) 2 : 9 b) 9 : 2 c) 7 : 2 d) 2 : 5
- 10) A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, -5) is the mid-point of PQ, then the coordinates of P and Q are, respectively
 a) (0, -5) and (2, 0) b) (0, 10) and (-4, 0) c) (0, 4) and (-10, 0) d) (0, -10) and (4, 0)
- 11) Find the value of a + b + c, such that the numbers a, 10, b, c, 31 are in A.P./
 If following numbers are in AP: a, 7, b, 23, c, then Evaluate : a - 2b + c. (BONUS)

12) The areas of two similar triangles are 49 cm^2 and 64 cm^2 respectively. If the difference of the corresponding altitudes is 10 cm, then the lengths of altitudes

13) A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter 'l' of the hemisphere is equal to the edge of the cube. Surface area of the remaining solid.....

14) Find the value of k such that polynomial $x^2 - (k+6)x + 2(2k-1)$ has sum of its zeroes is equal to half of their product .

OR

If the roots of the equation $12x^2 + mx + 5$ the ratio 3:2, then m

15) Two friends were born in year of 2010. What is probability that they have the same birthday ?

a) $\frac{1}{365}$

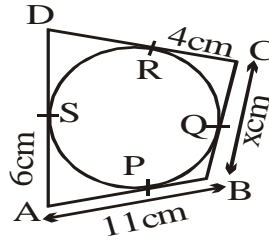
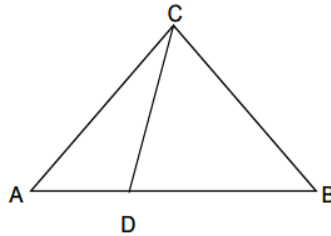
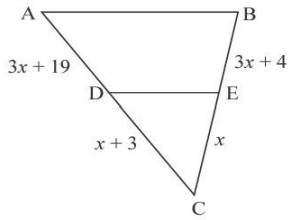
b) 0

c) $\frac{1}{366}$

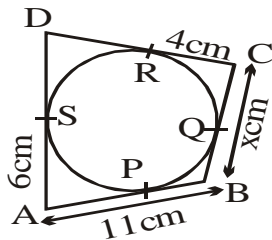
d) 1

16) In figure, find the value of x which will make DE || AB?

In the figure, if $\angle ACB = \angle CDA$, AC = 6 cm and AD = 3 cm, then find the length of AB. (BONUS)



17) In the given figure, the circle touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R, S respectively. If AB = 11 cm, BC = x cm, CR = 4 cm and AS = 6 cm, find the value of x.



OR

If an angle between two tangents drawn from an external point 'P' to a circle of radius r & centre O is 60° , Find the length of OP.

18) The sum of first n terms of an A.P. is given by $(n^2 + 3n)$. Find the sum of first term & common difference .

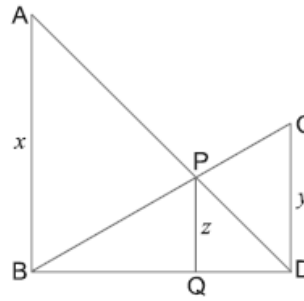
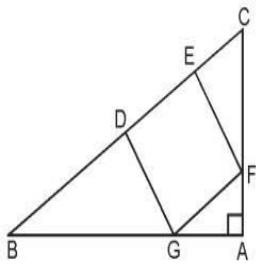
19) What can you say about the product of rational & irrational number , justify your answer . OR

Write rational & Irrational Number between $\sqrt{2}$ & $\sqrt{3}$ (BONUS)

20) If -2 is a root of the quadratic equation $x^2 - px - 5 = 0$ & quadratic equation $x^2 + px + k = 0$ has equal roots, find the value of k

SECTION -B

21) Prove that tangent at any point of circle is perpendicular to radius through point of contact, Hence prove that If a, b, c are the sides of a right triangle where c is the hypotenuse, then prove that radius r of the circle touches the sides of the triangle is given by $r = \frac{a+b-c}{2}$.



22) Find the middle term of sequence formed by all three digit numbers which leave a remainder 3, when divided by 4.

23) In Figure, DEFG is a square & $\angle BAC = 90^\circ$, Show that $DE^2 = BD \times EC$. OR

In the given figure, it is given that $\angle ABD = \angle DCB = \angle DPQ = 90^\circ$. If $AB = x$ units, $CD = y$ units and $PQ = z$ units, prove that

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

24) A tower is a tall structure, taller than it is wide, often by a significant margin. Towers are distinguished from masts by their lack of guy-wires and are therefore, along with tall apartment buildings, self-supporting structures. Towers are specifically distinguished from "apartment buildings" in that they are not built to be habitable but to serve other functions.

Karan went to city and he saw a transmission tower fixed at the top of a high building. He come to know that the height of the building is 20m. From a point on the ground, the angles of elevation of the top and the bottom of a transmission tower are α and β respectively such that $\cos\alpha = \sin(150^\circ - \alpha)$ and $\sin 2\beta = \cos(135^\circ - 3\beta)$. Find the height of the tower.

25) Two dice are thrown together. Find the probability that :

- i) $P(\text{a multiple of 2 on first die \& a multiple of 3 on second die})$
- ii) $P(\text{the sum of the numbers on the two faces is divisible by 4 \& 6})$
- iii) $P(\text{Getting 5 only once})$

OR

A bag contains white, black and red balls only. A ball is drawn at random from the bag. If the probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$, then find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag.

26) Three cubes of a metal whose edges are in the ratio 3:4:5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

SECTION – C

27) Prove that product of any three consecutive integers divisible by 6 .

OR

Given that $\sqrt{5}$ is an irrational number, Prove that $3 - 2\sqrt{5}$ is an irrational number.

28) If the ratio of sum of the first m and n terms of an A.P. is $m^2 : n^2$, show that the ratio of its m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$.

If Sum of first m terms of an AP is n & sum of first n terms is m , then show that the sum of first $(m + n)$ terms is $-(m+n)$. (BONUS)

29)i) A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water. OR

The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

ii) Solve : $\frac{21}{x} + \frac{47}{y} = 110$; $\frac{47}{x} + \frac{21}{y} = 162$, $x, y \neq 0$.

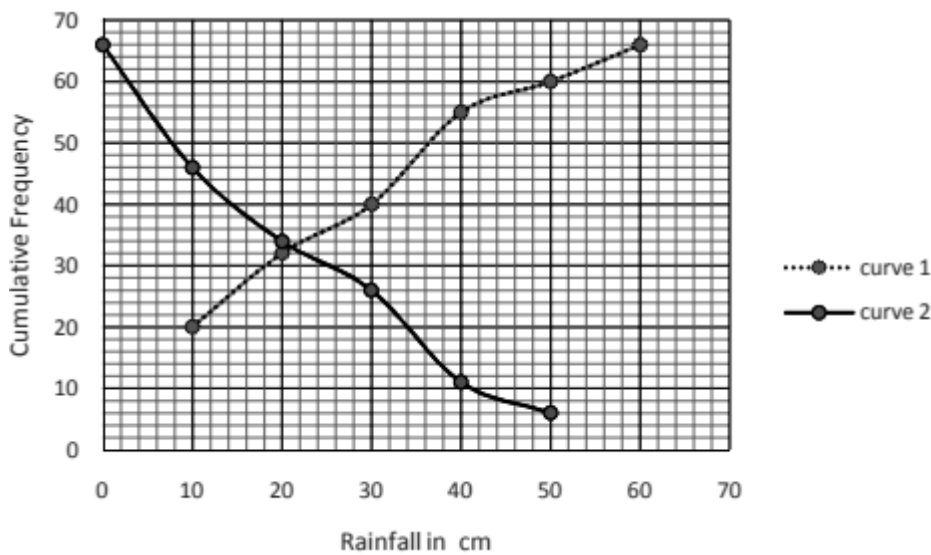
30) Prove that : $\frac{\sec^3 \theta}{\sec^2 \theta - 1} + \frac{\operatorname{cosec}^3 \theta}{\operatorname{cosec}^2 \theta - 1} = \sec \theta \operatorname{cosec} \theta (\sec \theta + \operatorname{cosec} \theta)$ OR

If $\sec A - \tan A = x$, Show that : $\frac{x^2 + 1}{x^2 - 1} = -\operatorname{cosec} A$.

31) Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials.

32) Sides of a right triangular field are 25m, 24m and 7m. At the three corners of the field, a cow, a buffalo and a horse are tied separately with ropes of 3.5 m each to graze in the field. Find the area of the field that cannot be grazed by these animals.

33) A TV reporter was given a task to prepare a report on the rainfall of the city Dispur of India in a particular year. After collecting the data, he analyzed the data and prepared a report on the rainfall of the city. Using this report, he drew the following graph for a particular time period of 66 days.

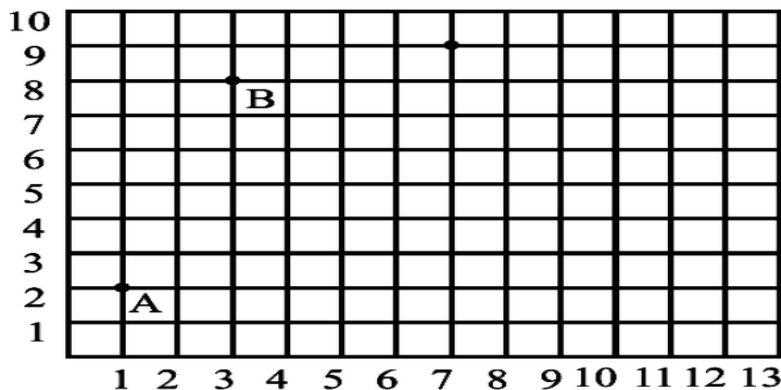


Based on the above graph, answer the following questions:

- (i) Identify less than type ogive and more than type ogive from the given graph.
- (ii) Find the median rainfall of Dispur
- (iii) Obtain the Mode of the data if mean rainfall is 23.4cm
- (iv) Make Frequency distribution table from above graph.

34) One day, three friends Aakash, Aditya and Manoj went to Children Park to play some games.

While playing at one moment Aakash is at the point P, Aditya is at point A and Manoj is at point B such that Aakash's distance from Aditya and Manoj are equal. If the position of Aditya and Manoj are given as (1, 2) and (3, 8) respectively and area of triangle PAB = 10 sq. m, then find the coordinates of P. Also find the distance of Aakash from Aditya and Manoj.



SECTION –D

35) A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45° , how soon after this will the car reach the observation tower. Give your answer correct to nearest seconds.

36) Draw a triangle ABC with side $BC = 7$ cm, $\angle B = 45^\circ$, $\angle A = 105^\circ$. Then, construct a triangle whose sides are 1.6 times the corresponding sides of $\triangle ABC$.
OR
Draw pair of tangents to circle of radius 5 cm which are inclined at angle of 45° .

37) A farmer connects a pipe of internal diameter 25 cm from a Canal into a cylindrical tank in his field which is 12 m in diameter and 2.5 m deep if water flows through the pipe at the rate of 3.6 km per hour in how much time will the tank be filled also find the cost of water is the canal department charges @ rupees 0.07 per metre³.
OR

A milk container is made of metal sheet in the shape of frustum of a cone whose volume is $10459\frac{3}{7}$ cm³. The radii of its lower and upper circular ends are 8 cm and 20 cm respectively. Find the cost of metal sheet used in making the container at the rate of Rs 1.40 per square centimeter.

38) The distribution of heights in cm of 96 children is given below:

Height	124-128	128-132	132-136	136-140	140-144	144-148	148-152	152-156	156-160	160-164
Children	5	8	17	24	16	12	6	4	3	1

Draw a less than type & more than type ogive from given data, Hence obtain median from the graph.

39) In a right triangle ABC, right-angled at C, P and Q are points on the sides CA and CB respectively which divide these sides in the ratio 1 : 2. Prove that

(i) $9AQ^2 = 9AC^2 + 4BC^2$ (ii) $9BP^2 = 9BC^2 + 4AC^2$ (iii) $9(AQ^2 + BP^2) = 13AB^2$.

40) A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train & time taken.
OR

i) Solve : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

ii) A pole has to be erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Find the distances from the two gates where the pole is to be erected.