EXCEL EDUCATIONAL CENTRE, ALTHUMAMA

Class – XII

MATHEMATICS- 041

Model Examination 2019-20

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

(i) All the questions are compulsory.
(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION A

Q1 - Q10 are multiple choice type questions. Select the correct option.

1. If A and B are two matrices of the order $3 \times m$ and $3 \times n$, respectively, and $m = n$, then the order of matrix $(5A - 2B)$ is

   (a) $m \times 3$ (b) 3×3 (c) $m \times n$ (d) $3 \times n$

2. The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$

   (a) $a^3+b^3+c^3$ (b) $3abc$ (c) $a^3+b^3+c^3-3abc$ (d) none of these

3. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{\imath})^2+(\vec{a}\times\hat{\jmath})^2+\left(\vec{a}\times\hat{k}\right)^2$ is equal to

   (a) $\vec{a}^2$ (b) $3\vec{a}^2$ (c) $4\vec{a}^2$ (d) $2\vec{a}^2$

4. Let A and B two given events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$, then $P(A'/B')$ is

   (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$.

5. The maximum value of $4x+5y$ subject to the constraints $x+y\le 20,\ x+2y\le 35,\ x-3y\le 12$ is

   (a) 84 (b) 95 (c)100 (d)96

6. If $sin^{-1}x - cos^{-1}x = \frac{\pi}{6}$, then the value of x is

   (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{2}$.

7. Two events A and B will be independent, if

   (a) A and B are mutually exclusive (b) P(A′B′) = [1 – P(A)] [1 – P(B)]

   (c) P(A) = P(B) (d) P(A) + P(B) = 1

8. $\int \frac{dx}{\sin(x-a)\sin(x-b)}$ is equal to

   (a) $\sin(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$ (b) $\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + C$

   (c) $\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$ (d) $\sin(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + C$

9. Distance of plane $\vec{r}.\left(2\hat{\imath}+3\hat{\jmath}-6\hat{k}\right)+2=0$, from the origin is

   (a) 2 units (b) 14 units (c) $\frac{2}{7}$ units (d) None of these.

10. Distance of the point (α, β, γ) from y-axis is

(a) β (b) $|\beta|$ (c) $|\beta| + |\gamma|$ (d) $\sqrt{\alpha^2 + \gamma^2}$

(Q11 - Q15) Fill in the blanks

11. Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = 2x + 3$. Then $f^{-1}(x)$is equal to:_____.

12. The function $f(x) = \begin{cases} 1 + x & if\ x \le 2 \\ 5 - x & if\ x > 2 \end{cases}$ is not differentiable at $x =$ ___.

13. A square matrix A is said to be idempotent matrix, if _____.

14. The intervals in which the function f is given by $f(x) = x^2 - 4x + 6$ is ____.

OR

For the curve $y = (2x + 1)^3$, then the rate of change of slope at $x = 1$ is____.

15. The unit vector in the direction of the sum of the vectors $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ is _______.

OR

If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a}.\vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ is____.

(Q16 - Q20) Answer the following questions

16. For what value of x the matrix $\begin{bmatrix} 5 - x & x + 1 \\ 2 & 4 \end{bmatrix}$ is singular?

17. Evaluate: $\int_0^{\pi/2} \sqrt{1 - \sin 2x}\ dx$

18. Evaluate: $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$.

OR

Evaluate: $\int \frac{x^3 \sin\left(\tan^{-1}\left(x^4\right)\right)}{1 + x^8} dx$.

19. Evaluate: $\int_1^{\sqrt{3}} \frac{1}{1 + x^2} dx$.

20. Find the Integrating factor of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$.

SECTION B

21. Prove that: $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}\sqrt{x}$.

OR

What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

22. Find the differential equation of all circles which pass through the origin and whose centre lies on y-axis.

23. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on $\mathbb{R}$..

24. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$.

OR

Find $|\vec{a}|$and $|\vec{b}|$, if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$=8 and $|\vec{a}|=8|\vec{b}|$.

25. Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$ are perpendicular to each other.

26. Given that the events A and B are such that P(A) = 1/2 , P(A ∪ B) = 3/5 and P(B) = p. Find p if they are (i) mutually exclusive (ii) independent.

SECTION C

27. Show that the relation R defined by $(a,b)R\,(c,d) \Rightarrow a+d=b+c$ on $A\times A$, where $A=\{1,2,3,\dots,10\}$ is an equivalence relation. Hence write the equivalence class $[(3,4)]$; $a,b,c,d\in A$.

28. If $y=\log\left[x+\sqrt{x^2+1}\right]$, show that $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$.

OR

Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $x\in(-1,1), x\neq 0$.

29. Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}=\left(e^{m\tan^{-1}x}-y\right)$, given that $y=1$, when $x=0$.

30. Evaluate: $\int_0^a \left(\frac{x^4}{\sqrt{a^2-x^2}}\right)dx$.

31. Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

OR

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X=x)=\begin{cases}0.1, \text{ if } x=0\\ kx, \text{ if } x=1 \text{ or } 2\\ k(5-x), \text{ if } x=3 \text{ or } 4\\ 0, \text{ otherwise}\end{cases}$$

(a) Find the value of k.

(b) What is the probability that you study at least two hours? Exactly two hours? At most two hours?

32. A merchant plans to sell two types of personal computers − a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

SECTION D

33. Prove that $\begin{vmatrix} x & x^2 & 1+px^3\\ y & y^2 & 1+py^3\\ z & z^2 & 1+pz^3\end{vmatrix}=(1+pxyz)(x-y(y-z)(z-x)$.

OR

Given $A=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}, B=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$, find BA and use this to solve the system of equations:
$y+2z=7, \quad x-y=3, \quad 2x+3y+4z=17.$

34. Using integration, find the area of the region $\{(x, y): x^2 + y^2 \leq 2ax,\ y^2 \geq ax, x, y \geq 0\}$

35. Show that the semi-vertical angle of a right circular cone of a given surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.

OR

Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$with its vertex at one end of the major axis.

36. Find the equation of the plane through the line of intersection of the planes $\vec{r}\,.\left(2\hat{\imath} - 3\,\hat{\jmath}\ + 4\,\hat{k}\ \right) = 1$ and $\vec{r}\,.(\,\hat{\imath} - \,\hat{\jmath}\,) + 4 = 0$ and which is perpendicular to the plane $\vec{r}\,.\left(2\hat{\imath} - \hat{\jmath} + \ \hat{k}\ \right) + 8 = 0$. Hence find whether the plane thus obtained contains the line $x - 1 = 2y - 4 = 3z - 12$.

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